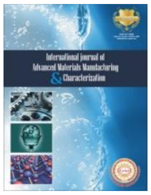

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Development of Modified Arrhenius Model for Ti-6al-4v Alloy to Predict the Flow Stress

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A B S T R A C T

Strain, strain rate and temperature have a significant impact on the flow stress of a material. To study the impact of these factors on flow stress, experiments are conducted at various strain rates (0.1 - 0.0001s^{-1}) and at various temperatures (323K, 348 K and 373K) on Ti-6Al-4V alloy. Stress values are taken for the corresponding strain values at an interval of 0.01(0.01- 0.08). It is observed that the flow stress is a function of strain, strain rate and temperature. The sine-hyperbolic law in an Arrhenius-type equation has been successfully applied for prediction of flow behavior of materials. The original model has been revised several times to suitably represent the flow behavior of various grades of materials. In this model, an exponential strain-dependent parameter was introduced in the sine-hyperbolic constitutive equation to predict the flow stress. The combined effect of the temperature and strain rate on the deformation behavior is represented by the Zener–Hollomon parameter (Z) in an exponent-type equation. It is observed that the predicted flow stresses are in good agreement with the experimental data.

Introduction

Metal forming is the backbone of modern manufacturing industry besides being a major industry in itself. Metal forming is a general term, for a large group, that includes a wide variety of manufacturing processes. Metal forming processes are characteristic in that the metal being processed is plastically deformed in order to shape it into a desired geometry. In order to plastically deform a metal, a force must be applied that will exceed the yield strength of the material. Properties of metal get altered with an increase in temperature. Therefore the metal will react differently to the same manufacturing operation if it is performed under different temperatures, and the manufactured part may possess different properties. For these reasons it is very important to understand the materials behavior at high temperature [1, 2].

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Ti-6Al-4V alloy presently is the most widely used alloy, accounting for more than 50% of all titanium tonnage in the world [1]. It is designed primarily for high strength at low to moderate temperatures; it has a high specific strength (strength/ density), stability at temperature up to 400^{0}c and good corrosion resistance [2]. Ti-6Al-4V alloy is used extensively for turbine engines, airframe, engine components include blades, discs, wheels. In addition, the superplastic characteristics of fine grained, equiaxed Ti-6Al-4V alloy is being used increasingly for aerospace applications. It also has good diffusion-bonding characteristics, which combined with superplastic forming, enables the fabrication of very complex structures. Its exceptional corrosion/erosion resistance provides the prime motivation for chemical process, marine use, food processing plants, oil refinery heat exchangers, medical prostheses and industrial use.

Today, titanium alloys are common, readily available engineered metals that compete directly with stainless and specialty steels, copper alloys, nickel based alloys and composites [3, 4]. Titanium is widely being used in critical applications which operate at high temperature applications like structural elements of civilian and military aircrafts and nuclear components[5, 6]. This necessitates the need to understand the mechanisms of deformation and the relationship between the processing variables, microstructure, and properties In this work experiments are conducted at various strain rates (0.1-0.0001s-1) at various temperatures(323K, 348K and 373K) on Ti-6Al-4V alloy. Stress values are taken for the corresponding strain values at an interval of 0.01 (0.01- 0.0.08). Arrhenius – type equation [7] expresses the correlation between flow stresses, temperature and strain rate especially at elevated temperatures. Zener – Hollomon parameter in an exponential type equation represents the combined effects of temperature and strain rates on deformation behavior. In this research work modified-Arrhenius type model is developedtounderstand the deformation behavior of Ti-6Al-4V alloy. This model has 10 material constants, which are determined by applying regression analysis on the experimental data.

Experimental details

Flat tensile test specimens of Ti-6Al-4V alloy with 0.89 mm thickness were fabricated as per the Defence Metallurgical Research Laboratory, Hyderabad (DMRL)standards, which is a sub-sized version of ASTM: E8/E8M-11standardsas shown in Figure 1. Material composition of the alloy is as per the table 1. The samples were machined out of the raw titanium sheet by wire cutting electro-discharge machining process for high accuracy and finish. Laboratory tensile tests were carried out on a computer controlled servo-hydraulicUniversal Testing Machine (UTM) as shown in Figure 2, which has a maximum load capacity of 100kN. The machine is equipped with a controlled system to impose exponential increase of the actuator speed to obtain constant true strain rates and also a resistance heating split furnace which is used to heat the tensile test specimen up to 1000^{0}C. The pull rods for the high temperature testing at UTM are designed to be made of nickel base super alloy CM-247. The experiments were conducted at temperatures of 323K, 348K and 373K and at strain rates of 0.1, 0.01, 0.001 and 0.0001s^{-1} and the data had been collected. Standard equations were used to convert the load-displacement data to true stress-true strain data. The elastic region was subtracted from the true stress–strain curve to get true stress-true plastic strain data.

Table 1: Material composition of Ti-6Al-4V Alloy (wt. %).

Element	AL	V	Fe	C	Ti
Comp%	5.56	4.07	0.185	0.022	89.997

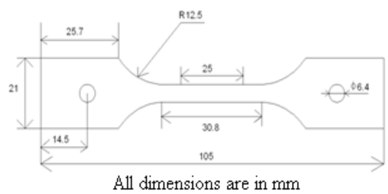

Figure 1: Flat tensile specimen as per DMRL standards

Figure 2: Computer controlled UTM of 100kN capacity with resistance heating three zone furnace

Modified–arrhenius type constitutive equation:

Arrhenius – type equation [7, 8] expresses the correlation between flow stresses, temperature and strain rate especially at elevated temperatures. Zener – Hollomon parameter in an exponential type equation represents the combined effects of temperature and strain rates on deformation behavior. These equations can be mathematically expressed as:

$$Z = \dot{\varepsilon}\exp(\frac{Q}{RT}) \quad (1)$$

$$\dot{\varepsilon} = AF(\sigma)\exp(-\frac{Q}{RT}) \quad (2)$$

Where,

$$F(\sigma)\begin{cases} \sigma^n \alpha\sigma < 0.8 \\ \exp(\beta\sigma)\,\alpha\sigma > 1.2 \\ [\sinh(\alpha\sigma)]^n for all \sigma \end{cases} \quad (3)$$

R is universal gas constant (8.314 J mol^{-1} K^{-1}), T is absolute temperature (K), σ is flow stress in MPa and Q is activation energy of hot deformation in kJ mol^{-1}. A, α and n are material constants, α= β/n.

Substituting$F(\sigma) = [\sinh(\alpha\sigma)]^n$ into (2) gives an ameliorated equation in a hyperbolic sinusoidal form, which describes dependency of steady – state flow stress on temperature and strain rate. Final equation after substitution is as follows:

$$\dot{\varepsilon} = A[\sinh(\alpha\sigma)]^n \exp(-\frac{Q}{RT}) \quad (4)$$

Flow stress can be written as a function of Zener – Hollomon parameter according to hyperbolic law.

$$\sigma = \frac{1}{\alpha}\ln\left\{\left(\frac{Z}{A}\right)^{1/n} + \left[\left(\frac{Z}{A}\right)^{2/n} + 1\right]^{1/2}\right\} \quad (5)$$

But the effect of strain is not taken into account by this constitutive equation. To consider this effect, Xiao and Guo [9] expressed the relationship between strain and flow stress as follows:

$$\sigma = \beta_0 \varepsilon^{\beta_1} \exp(-\beta_2 \varepsilon) \qquad (6)$$

Where, β_0, β_1 and β_2 are constants.

$$\sigma = \frac{\beta_0}{\alpha} \varepsilon^{\beta_1} \exp(-\beta_2 \varepsilon) \ln\left\{\left(\frac{Z}{A}\right)^{1/n} + \left[\left(\frac{Z}{A}\right)^{2/n} + 1\right]^{1/2}\right\} \qquad (7)$$

The material constants A,α, n, Q, β_0, β_1 and β_2 are determined by using the stress – strain data from the experiments done under different deformation temperatures and strain rates.

Now, final constitutive equation which satisfactorily describe the effect of strain rate, temperature and strain on steady – state flow stresses, is developed by combining (5) and (6) as follows:

Development of the model

By substituting the power law and exponential law of $F(\sigma)$ into (2) following relationships are obtained for low stress levels ($\alpha\sigma < 0.8$) and high stress levels ($\alpha\sigma > 1.2$), respectively, $\dot{\varepsilon} = A_1 \sigma^{n_1}$ and $\dot{\varepsilon} = A_2 \exp(\beta\sigma)$ where,A_1, A_2 and n_1 are material constants independent of experimental temperature. Taking logarithm on both sides of these equations and plotting $\ln \dot{\varepsilon}$ vs $\ln \sigma$ and$\ln \dot{\varepsilon}$ vsσ by substituting the values of flow stress and its corresponding strain rate under the strain of 0.01 to 0.08 and various temperatures, the relationships between flow stress and strain rate are obtained. n_1 and β can be calculated as mean slopes of the graphs. n_1 and β are 22.5412 and 0.0047 MPa^{-1} respectively, which gives value of $\alpha = \beta/n_1$ = 2.0932*10^{-4} MPa^{-1}.

For all stress levels taking logarithm on both sides of (4), yields,

$$ln[sinh(\alpha\sigma)] = \frac{1}{n} ln\, \dot{\varepsilon} + \frac{Q}{nRT} - \frac{1}{n} ln\, A \qquad (8)$$

The value of material constant n can be obtained as mean slopes of lines in $ln[\sinh(\alpha\sigma)]$ vs. $ln\, \dot{\varepsilon}$ plot, which is plotted by taking values of strain rate and flow stress at different strains and particular temperature and value of n was found to be 131.50.Q denotes activation energy and reflects the deformation difficulty degree in hot deformation. Now differentiating (8) at particular strain rate, yields,

$$Q = Rn \frac{d(ln[\sinh(\alpha\sigma)])}{d(1/T)} \qquad (9)$$

The value of Q, i.e. slope of $ln[\sinh(\alpha\sigma)]$ vs. $1/T$ plot multiplied by R*n, is calculated for each strain and strain rate. Since the experiments have been conducted at 4 strain rates and 8 strain values are considered, which give a total 32 values of Q. Now, average of these Q values is 519.55kJ mol^{-1}.

Performing nonlinear regression analysis on experimental data obtained from 12 sets of experiments gives the values of constants β_0, $\beta_1$and β_2 as shown in Table 3, 4 and 5 respectively. The values of β_0, $\beta_1$and β_2 are not constant and change within a small extent due to change in strain rate and deformation temperature. Combined effects of temperature and strain rate on these constants can be represented by Zener – Holloman parameter. It is observed that these constants and lnZ follow a linear relationship as represented by following equations:

Zener – Hollomon parameter expresses effects of deformation temperature and strain rate on flow stress. Now substituting (2) into (4) and taking logarithm on two sides yields the following expression of Zener – Hollomon parameter:

$$ln\, Z = ln\, A + n ln[sinh(\alpha\sigma)] \qquad (10)$$

Plot of $ln[\sinh(\alpha\sigma)]$ vs. lnZ at a particular strain gives the value of *ln A.* Now $ln[\sinh(\alpha\sigma)]$ vs. lnZ is plotted for all 8strain values and the average value of lnA is calculated as 305.659. Thus, the value of A comes out to be 5.5719*$10^{132}$$s^{-1}$.

Table 2: Material Constants for Titanium Grade 5 (Ti-6Al-4V) alloy

Parameter	α(MPa^{-1})	n	Q (kJ/mol)	A (s^{-1})
Value	2.0932*10^{-4}	131.50	519.55	5.5719*10^{132}

The material constants Q, A, n andα are listed in Table 2. Using these material constants the combined effect of deformation temperature and strain rate on flow stress at a particular strain can be described by following equation:

$$\sigma = \frac{1}{2.0932*10-4} ln\left\{\left(\frac{Z}{5.5719*10132}\right)^{1/131.50} + \left[\left(\frac{Z}{5.5719*10132}\right)^{2/131.50} + 1\right]^{0.5}\right\} \qquad (11)$$

where$Z = \dot{\varepsilon} \exp\left(\frac{519.55\times10^3}{8.314T}\right)$

Now, non – linear regression method as mentioned in [7] is used to determine the unknown parameters β_0, β_1 and β_2 called regression coefficients. Treating (7) as the fitting model, where the strain (ε) denotes independent variable, fitting model can be mathematically expressed as:

$$y = \frac{\beta_0}{\alpha} x^{\beta_1} \exp(-\beta_2 x) \ln\left\{\left(\frac{Z}{A}\right)^{1/n} + \left[\left(\frac{Z}{A}\right)^{2/n} + 1\right]^{1/2}\right\} \qquad (12)$$

Denoting flow stress calculated from (11) by σ_z and putting it in (12) simplifies it as:

$$y = \beta_0 x^{\beta_1} \exp(-\beta_2 x)\sigma_z \qquad (13)$$

$$\left.\begin{aligned} \beta_0 &= 0.0055\,.\,lnZ - 0.2030 \\ \beta_1 &= 0.0011\,.\,lnZ - 0.1288 \\ \beta_2 &= 0.0568\,.\,lnZ - 9.5577 \end{aligned}\right\} \qquad (14)$$

Table 3: β_1 values

$\dot{\varepsilon}/s^{-1}$	0.0001	0.001	0.01	0.1
323	0.68	0.76	0.73	0.71
348	0.64	0.73	0.84	0.69
373	0.60	1.14	0.70	0.88

Table 4: β_2 values

$\dot{\varepsilon}/s^{-1}$	0.0001	0.001	0.01	0.1
323	0.044	0.063	0.045	0.056
348	0.057	0.056	0.077	0.05
373	0.041	0.13	0.054	0.077

Table 5: β_3 values

$\dot{\varepsilon}/s^{-1}$	0.0001	0.001	0.01	0.1
323	-0.096	-0.068	-0.372	-0.604
348	-0.435	-0.180	1.46	-0.018
373	-0.276	3.77	0.249	0.882

Results and discussion

The equation for Modified- Arrhenius type model is given by

$$\sigma = \frac{\beta_0 \varepsilon^{\beta_1} \exp(-\beta_2 \varepsilon)}{2.0932*10-4} ln\left\{\left(\frac{Z}{5.5719*10132}\right)^{1/131.50} + \left[\left(\frac{Z}{5.5719*10132}\right)^{2/131.50} + 1\right]^{0.5}\right\} \quad (15)$$

Where,

$$\left.\begin{aligned} \beta_0 &= 0.0055 \,.\, lnZ - 0.2030 \\ \beta_1 &= 0.0011 \,.\, lnZ - 0.1288 \\ \beta_2 &= 0.0568 \,.\, lnZ - 9.5577 \end{aligned}\right\} \quad (16)$$

And Z is given by

$$Z = \dot{\varepsilon} \exp\left(\frac{519.55\times10^3}{8.314T}\right) \quad (17)$$

Figures 3, 4, 5 and 6 show the plots between flow stress and true plastic strain values at different temperature and strain rates for modified- Arrhenius model. The correlation coefficient for this model is found to be 0.8405 as shown in Figure 7 and Δ came out to be 5.1859% with a standard deviation of 3.7659.The correlation coefficient is low but the error value is significantly less.The error value is less which means it can be acceptable for prediction of flow stress.

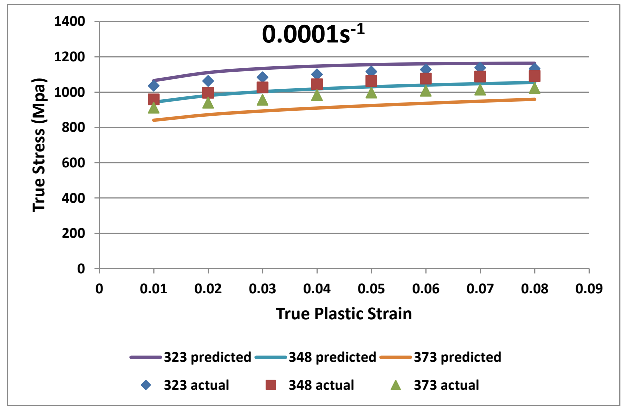

Figure 3: Graph showing experimental and predicted stress for modified Arrhenius Model at various strains for strain rate of $0.0001s^{-1}$

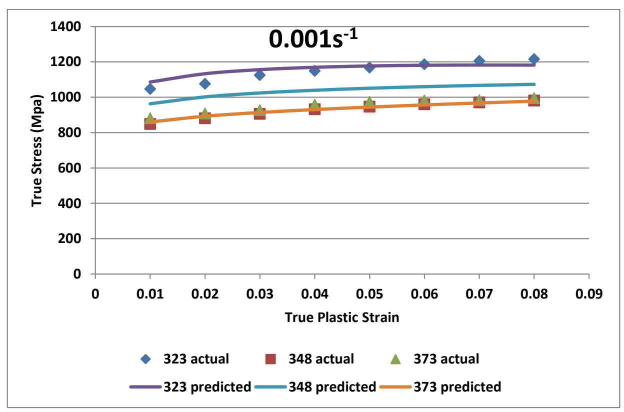

Figure 4: Graph showing experimental and predicted stress for modified Arrhenius Model at various strains for strain rate of $0.001s^{-1}$

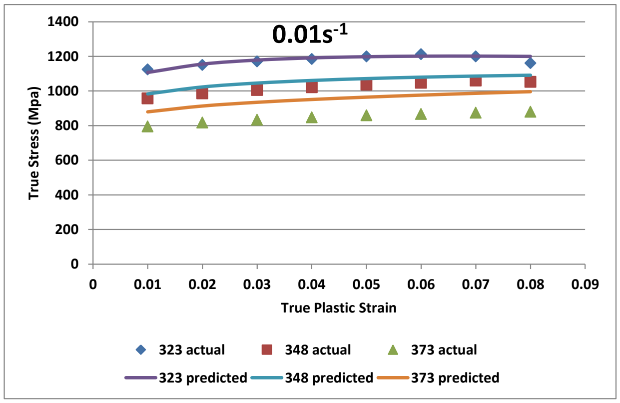

Figure 5: Graphs showing experimental and predicted stress for modified Arrhenius Model at various strains for strain rate 0.01 s^{-1}.

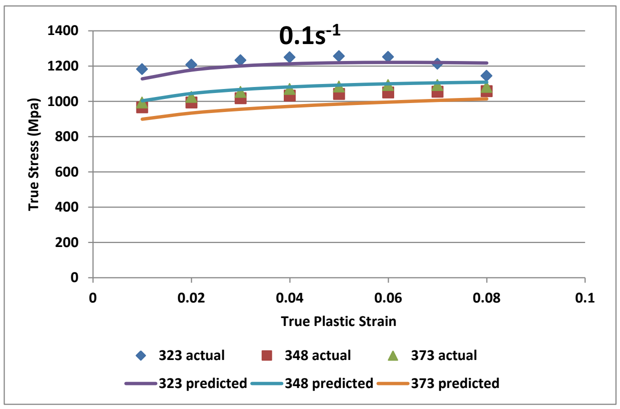

Figure 6: Graphs showing experimental and predicted stress for modified Arrhenius Model at various strains for strain rate 0.1 s^{-1}.

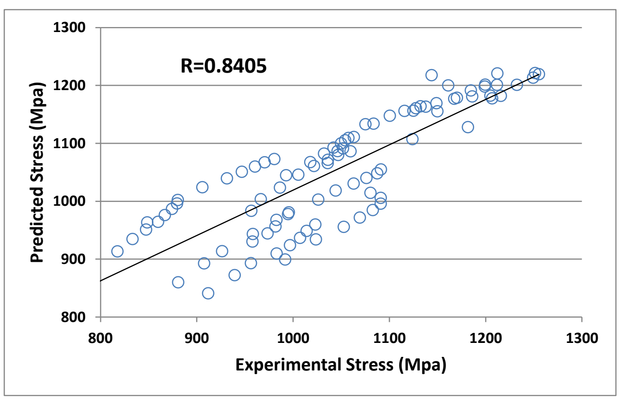

Figure 7: Plot of predicted vs experimental stress for Modified Arrhenius Model.

Conclusion

The value of flow stress depends on three parameters, namely, strain, strain rate and temperature. The prediction of modified Arrhenius model is more accurate at reference temperature and strain rate. But as the temperature and strain rate increases, the error increases. Hence, it can be concluded that the modified Arrhenius model is suited for low temperature and low strain rate data. The error value is less which means it can be used for prediction of flow stress of Ti-6Al-4V alloy.

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