

# Advanced Materials Manufacturing & Characterization

journal home page: [www.ijammc-griet.com](http://www.ijammc-griet.com)



## Numerical Evaluation of Stress Intensity Factor for Inclined-Edge Crack Geometry using Singularity Elements

Nikhil Tatke, Nitin Kotkunde

<sup>1</sup>Department of Mechanical Engineering, BITS-Pilani, Hyderabad Campus, Hyderabad – 500078

### ARTICLE INFO

#### Article history:

Received 1 Nov 2012

Accepted 26 Dec 2012

#### Keywords:

Inclined edge crack,  
stress intensity factor,  
Singularity elements

### ABSTRACT

Edge cracks are more dangerous than interior cracks. Free edge close to the crack influences the stress field near the crack tip (since the free edge is traction free). In case of edge crack, the free edge is not only close to the crack, but it intersects the crack. Evaluation of Stress Intensity factor for oblique edge crack geometry is done using commercial FEM software ANSYS and compared with analytical results. The conventional elements always underestimate the sharply rising stress-displacement gradients near the crack tip. Therefore, in order to produce this singularity in stresses and strains Barsoum elements were employed which involves shifting the mid-side nodes to the quarter point locations. It observed that stress intensity factor found by FEM method has good agreement with analytical results..

### Introduction

The singular order of stresses near the interface is a good way of understanding of failure initiation; however, in engineering applications usually the knowledge of singular orders is not enough for the prediction of failure initiation. As an example in the case of homogeneous cracks, the singular order is  $-1/2$  which remains constant irrespective of the surrounding environment and outside loading of the crack. These influential factors are reflected through the associated stress intensity factor of the cracks. Hwu and Kuo [1] have demonstrated several different kinds of examples such as cracks in homogenous isotropic or anisotropic materials, central or edge notches in isotropic materials, interface cracks and interface corners between two dissimilar materials. It is also shown that KI Evaluation Using Displacement extrapolation technique under adaptive dense mesh with Parallel Finite Element gives fairly accurate results [2]. Based upon the analytical solutions obtained previously for the multi-bonded anisotropic wedges and the well-known path-independent H-integral, Hwu and Kuo [3] provided a unified definition and a stable computing approach for the stress intensity factors of interface corners.

The stress intensity factor calculations are usually limited to Linear Elastic Fracture Mechanics (LEFM). For a linear elastic material the stress and strain fields ahead of the crack tip are expressed as [4].

The singular order of stresses near the interface is a good way of understanding of failure initiation; however, in engineering applications usually the knowledge of singular orders is not enough for the prediction of failure initiation. As an example in the case of homogeneous cracks, the singular order is  $-1/2$  which remains constant irrespective of the surrounding environment and outside loading of the crack. These influential factors are reflected through the associated stress intensity factor of the cracks. Hwu and Kuo [1] have demonstrated several different kinds of examples such as cracks in homogenous isotropic or anisotropic materials, central or edge notches in isotropic materials, interface cracks and interface corners between two dissimilar materials. It is also shown that KI Evaluation Using Displacement extrapolation technique under adaptive dense mesh with Parallel Finite Element gives fairly accurate results [2]. Based upon the analytical solutions obtained previously for the multi-bonded anisotropic wedges and the well-known path-independent H-integral, Hwu and Kuo [3] provided a unified definition and a stable computing approach for the stress intensity factors of interface corners.

The stress intensity factor calculations are usually limited to Linear Elastic Fracture Mechanics (LEFM). For a linear elastic material the stress and strain fields ahead of the crack tip are expressed as [4].

- Corresponding author: Nikhil Tatke
- E-mail address: [nikhiltatke@gmail.com](mailto:nikhiltatke@gmail.com)
- Doi: <http://dx.doi.org/10.11127/ijammc.2013.02.026>

Copyright©GRIET Publications. All rights reserved.

$$\sigma_{i,j} = -K/(r^{0.5}) \times f_{ij}(\theta) \quad (1)$$

$$\epsilon_{i,j} = -K/(r^{0.5}) \times g_{ij}(\theta) \quad (2)$$

Where, K = stress intensity factor

(r, θ) = coordinates of a polar coordinate system used to describe the location of a point.

Very few literatures are available to comparative analysis of stress intensity factor (SIF) by using finite element analysis (FEM) and analytical results. In this paper, the commercial FEM software ANSYS is employed to calculate stress intensity factor for inclined edge crack geometry and the results obtained are compared with analytical method. Under plane strain assumption, the computation of the stress intensity factors is done by considering a strip with an inclined edge crack which is 1 in long.

### Modelling a crack-tip region

A fracture analysis is a combination of stress analysis and fracture mechanics parameter calculation. Computation of stress intensity factor using finite element analysis requires either a refined mesh around a crack tip or the use of some special elements with embedded stress singularity near the crack tip. It is worth noted that the conventional elements always underestimate the sharply rising stress-displacement gradients near the crack tip.

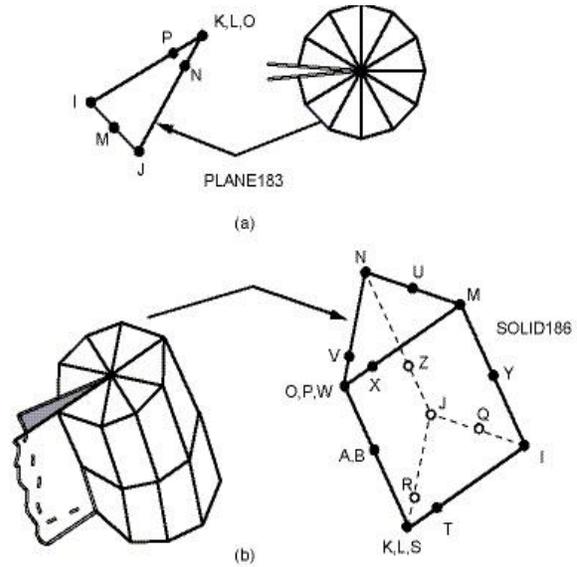
Instead of trying to capture the well-known  $1/(r^{0.5})$  singular behaviour with smaller and smaller elements, Barsoum introduced a direct method by shifting the mid-side node of an 8-noded isoparametric quadrilateral element to the one-quarter point from the crack tip node. Relocating the mid-side nodes to the one quarter point achieves the desired  $1/(r^{0.5})$  singular behaviour. For these locations of the mid nodes, the Jacobian becomes singular at the corner node, thus making displacement derivatives infinite and consequently stresses and strains become infinite as well [5].

In the case of linear elastic deformation, the elements Plane2 (2D, 6 noded triangle), Plane82 (2D, 8 noded quadrilateral), and Solid95 (3D, 20 noded brick) are employed. In order to produce this singularity in stresses and strains, the crack tip mesh should have certain characteristics which are mentioned below :

- The crack faces should be coincident.
- The elements around the crack tip (or crack front) should be quadratic, with the mid-side nodes placed at the quarter points. (Such elements are called singular elements.)

Following menu path is used in order to specify a key point to be the crack tip so that the elements around it will have singular stress capability,

Main Menu > Preprocessor > Meshing > Size Cntrls>Concentrat KPs > Create



(a) 2-D models and (b) 3-D models

Figure 1 - Examples of singular elements

For reasonable results, the first row of elements around the crack tip should have a radius of approximately  $a/8$  or smaller, where  $a$  is the crack length.

### Determination of Stress intensity factor

The Post1 KCALC command calculates the mixed-mode stress intensity factors  $K_I$ ,  $K_{II}$ , and  $K_{III}$ . However, this command has limitations that it can be used only in linear elastic problems with homogeneous, isotropic material around the crack tip region.

Following is the general process for calculating the stress-intensity factors:

Step 1: Model Generation

Step 2: Define a local coordinate system aligned with the crack faces. This coordinate system must be the active model coordinate system and results coordinate system when the KCALC executes.

Step 3: Define a path along the crack face in the vicinity of the crack tip. It is to be ensured that the first node in the path should be the crack tip node. For a half crack model, two additional nodes are required both along the crack face. In a full crack model both the crack faces are included. Therefore, two nodes are required at the top face and two nodes at the bottom face and in this order.

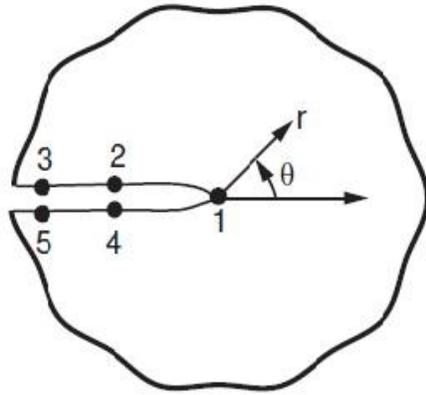


Figure 2 - Crack face path definition for a full-crack model

Step 4: Calculation of stress intensity factor (KCALC command) using the following menu driven path,

Main Menu > General PostProc > Nodal Calcs > Stress IntFactr

Here Plane strain condition with full crack model was considered for the analysis.

Step 5: Specify output controls and Post-processing the results.

### SIF determination for inclined-edge crack geometry using ANSYS

The geometry of the specimen with an inclined edge crack is as shown in figure 3. Plane strain assumption was used because in available literature, the values of critical SIFs of commonly used materials are given for plane strain condition. The reason being the critical SIF becomes a property of the material only for plane strain cases [6].

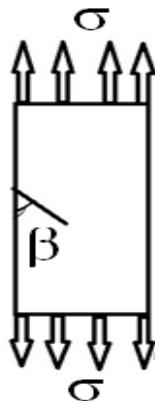


Figure 3 - Geometry of the strip with inclined edge crack

In this paper, we consider that the crack is 1 inch long and has an inclination of  $\beta$ . The width and length of the strip are 5 in. and 25 in. with the top and bottom surfaces being subjected to a tensile load of 100 psi. Other material properties are Young's Modulus,  $E = 30 \times 10^6$  psi and Poisson's Ratio  $\nu = 0.3$ .

Following figures illustrate the various steps used for obtaining stress intensity factor:

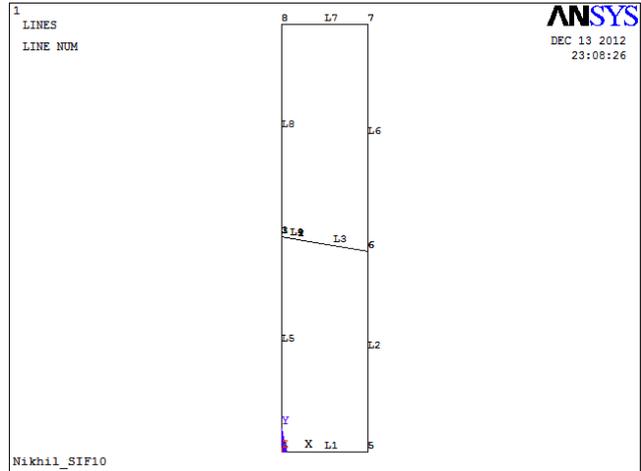


Figure 4 - First Step illustrating the line plot of the generated model indicating both keypoints and line numbers

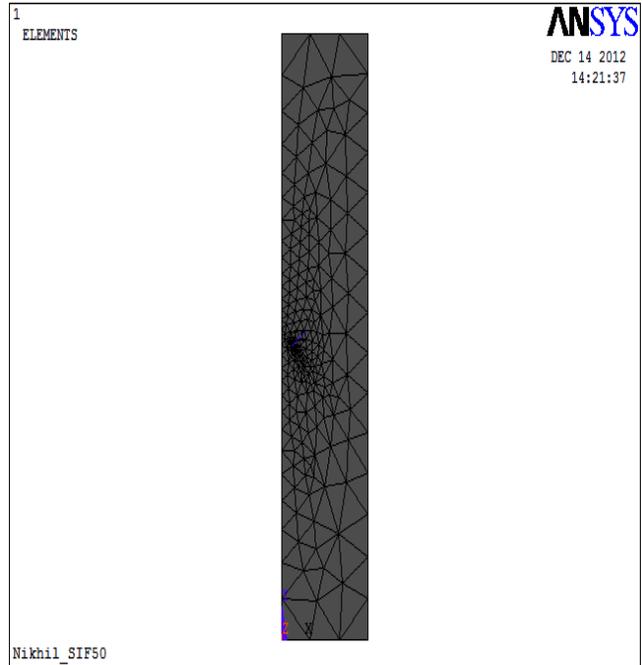


Figure 5 - Figure showing meshed areas of the strip

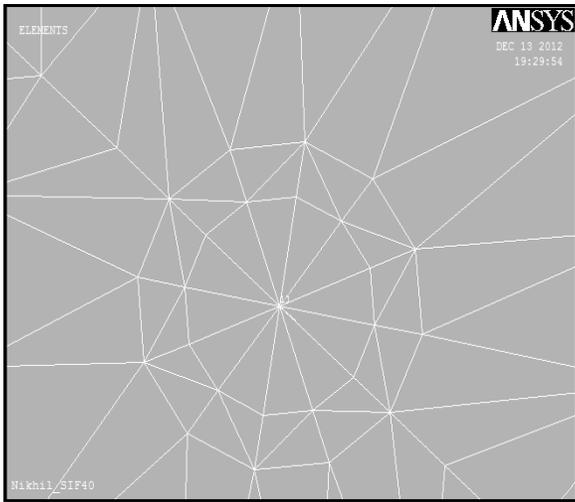


Figure 6 - Figure showing mesh pattern around the crack tip

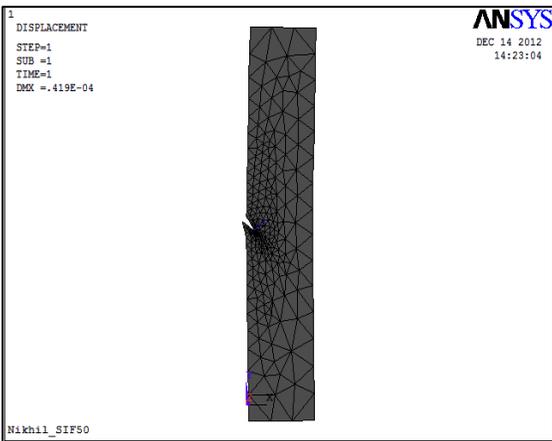


Figure 7 - Deformed shape of the strip



Figure 8 - Deformed shape in the vicinity of the crack tip

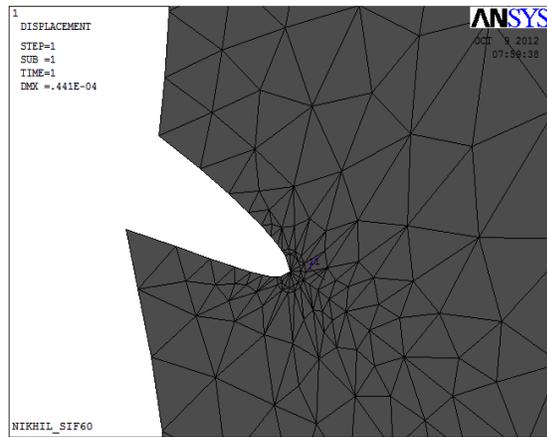


Figure 9 - Mesh Pattern in the vicinity of crack tip

Following are the comparative plots obtained between SIF's obtained by analytical method as well as that obtained by FEM simulation for different crack inclinations.

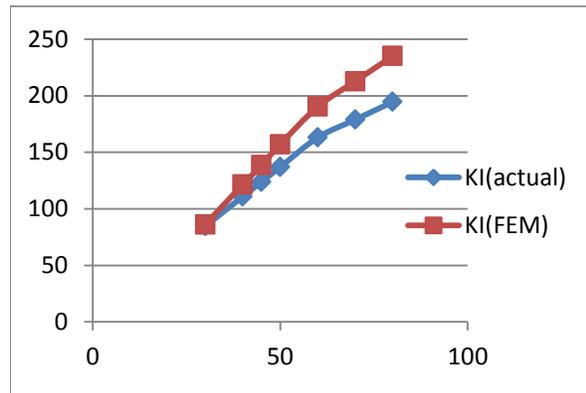


Figure 10 - Comparative plot for  $K_I$  at different crack inclinations

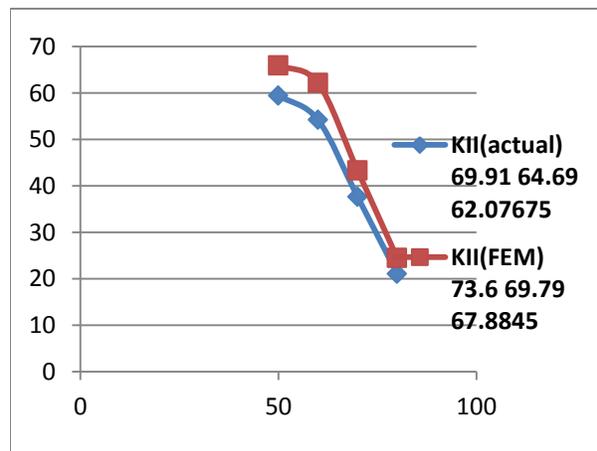


Figure 11 - Comparative plot for  $K_{II}$  at different crack inclinations

## Conclusion

The various edge crack geometries having different angle of inclinations have been analysed in commercial FEM software ANSYS. The stress intensity factors for both Mode I and Mode II have been calculated and the results compared with the analytical solutions. The results suggest that for a Mode I fracture as the angle of inclination increases difference between the two results increases while for Mode II fracture the results become almost equal. It has been found that SIF found by this method is fairly accurate.

## References

1. Hwu C., Kuo T.L., "A unified definition for stress intensity factors of interface corners and cracks", International Journal of solids and structures 44(2007) 6340-6359.
2. Hadi M.S.A., Ariffin A.K., "KI Evaluation Using Displacement Etrapolation Technique under Adaptive", IEEE (2008).
3. Hwu C., Kuo T.L., "Stress Intensity factors of interface corners", 2005 International Symposium on Electronics Materials and Packaging (EMAP2005), December 11-14, 2005.
4. Anderson, T. L., "Fracture Mechanics - Fundamentals and Applications", 2nd ed. Boca Raton: CRC, 1994.
5. Barsoum, R.S., "On use of Isoparametric Elements in Linear fracture Mechanics", 1976, International Journal for Numeric Methods in Engineering, 10, p. 25-38.
6. Murakami Y. "Stress Intensity Factors Handbook", Pergamon Press, Oxford.

