Multi Response Optimization For Correlated Responses In EDM Using Principal Component Analysis

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Abstract

This study presents a new method to determine multi-objective optimal condition using Principal Component Analysis (PCA) based on Grey Relational Analysis (GRA) Method for Characteristics as Material Removal Rate (MRR), Tool Wear Rate (TWR) and Surface Roughness (Ra) on Electrical Discharge Machining (EDM). In this paper, an attempt has been made to machining the AISI P20 tool steel by using cylindrical copper electrode perform on EDM. A well-designed L27 orthogonal array based on the Taguchi method were conducted on input parameters as current, pulse on time, up-time, working time and inter electrode gap.

Introduction

Electrical discharge machining process is one of the most successful, non-traditional machining processes for machining newly developed high strength alloys with high degree of dimensional accuracy, electrically conductive material irrespective of its hardness with economical cost of production. In electrical discharge machining (EDM), it is important to select machining parameters for achieving optimal machining performance [1]. Electrical discharge machining (EDM) is based on thermoelectric energy between the work piece and an electrode and it is one of the best alternatives for machining an ever increasing number of high-strength, non-corrosion, and wear resistant materials.

EDM researchers have explored a number of ways to improve and optimize the MRR, TWR and SR like Dhar and Purohit [2] evaluates the effect of effect of MRR, TWR, SR with Process parameters taken in to consideration were the current (I), the pulse duration (T) Al-4Cu–6Si alloy–10 wt. % SiCpcomposites. Despite a range of different approaches, all the research work in this area shares the same objectives of achieving more efficient material removal coupled with a reduction in tool wear and improved surface quality. Many researchers have carried out experimental research works using Taguchi method [3] with the aim to understand the effect of various process parameters on different response variables and/or to optimize the process parameters.

Considerable researches have been carried out in recent time aiming to establish an objective method for solving multi-response optimization problems using Taguchi method. The goal of multi response optimization is to find out the settings of the input variables that can achieve an optimal compromise of the response variables. With this aim, several multi response optimization approaches, in this regard, usually found in engineering literature, are weighted signal-to-noise (WSN) ratio method [4] and multi response Signal-to-Noise (MRSN) ratio method [5]. It has been shown that the grey-based Taguchi technique can optimise the multi-response processes because of grey relational analysis is used for solving the complicated interrelationships among the multiple responses [6]. In order to consider the possible correlation between the responses, some researchers have proposed to make use of PCA [7, 8].

PCA is a multivariate technique for forming new uncorrelated variables through a linear composite of the original variables. The maximum number of new variables that can be formed is equal to the number of original variables. In this paper, principal component analysis PCA-based GRA approaches which can optimise multiple responses taking care of the possible correlations between the responses are applied to experimental data on the EDM processes. Tong and Wang [9] proposed the PCA-based GRA method. The main advantage of PCA is that once
the patterns in data have been identified, the data can be compressed, i.e., by reducing the number of dimensions, without much loss of information.

Experimental design

The experiment was conducted on die-sinking EDM machine with 30 mm diameter of cylindrical copper tool electrode and EDM oil as dielectric. The work material was AISI P20 tool steel which is a semi-circular shaped work piece material (100 mm diameter and 10 mm thickness). Machining was carried out for 60 min for each experimental run. In this study, the effect of five process parameters, namely, current (Ip), on-time (Ton), up-time (Tup), work time (Tw), and inter electrode gap (IEG) and some fixed parameters, have taken like duty cycle (tau), voltage (V), flushing pressure (Fp) and polarity of the EDM process that shown in Table 1 was studied on three performance characteristics (responses) i.e., MRR, TWR, SR. The MRR is calculated as the ratio of the difference of weight of the tool before and after machining to the machining time and density of the material. TWR is expressed as the ratio of the difference of weight of the tool before and after machining to the machining time and density of the material and the SR was measured with TalySurf (Model: Taylor Hobson, Surtronic 3+) with parameters cut-off length, Ln= 4 mm, Sample length, Lc=0.8 mm and filter=2CR ISO.

The experiment designed using Taguchi method, which is an optimisation methodology that uses an orthogonal array to study the entire parameter space with only a small number of experiments. Taguchi’s signal-to-noise (S/N) ratios are logarithmic functions of desired output and serve as objective functions in the optimisation process. Usually, there are three categories of performance characteristics in the analysis of the S/N ratio:

- **Lower-The-Better (LTB),**
  \[ \eta_{ij} = -10 \times \log_{10} \left( \frac{\sum_{k=1}^{n} y_{ijk}^2}{\sum_{k=1}^{n} 1} \right) \]  
- **Higher-The-Better (HTB),**
  \[ \eta_{ij} = -10 \times \log_{10} \left( \frac{\sum_{k=1}^{n} 1}{\sum_{k=1}^{n} y_{ijk}} \right) \]  
- **Nominal-The-Best (NTB),**
  \[ \eta_{ij} = 10 \times \log_{10} \left( \frac{y_{ij}}{x_{ij}} \right) \]

Where, n=number of repeated experiments, \( y_{ijk} \) = experimental value of \( j \)th response variable in \( i \)th trial at \( k \)th replication.

Basically, in this paper only two S/N ratios are used i.e., LTB for TWR and SR and HTB for MRR. The experimental layout of the S/N ratios for the three response variables with the final output are given in Table 4. The modelling is done by MINITAB16 software using L27 array with Principle Component Analysis (PCA) method. After applying PCA on S/N ratios, the eigenvalues and the corresponding eigenvectors obtained are shown in Table 2.

### Proposed optimization (GRA Method):

Grey relational analysis based on the grey system theory means that a system in which part of information is known and part of information is unknown i.e., from black through grey to white is used for solving the complicated inter-relationships among the multiple responses. Grey Relational Analysis (GRA) a normalization evaluation technique is extended to solve the complicated multi-performance characteristics optimization effectively.

#### Procedure

The value obtained in the PCA-based GRA method is simply called overall quality performance index (OQPI).

**Step 1.** Compute the SN ratio for each responses.

**Step 2.** To obtain uncorrelated principal component scores (PCS), conduct PCA on the S/N ratios corresponding to each trial by using Equation:

\[ PCS_i = a_{1i} \eta_{11} + a_{2i} \eta_{22} + \ldots + a_{pi} \eta_{pi} \]  

The coefficients of the \( l \)th component, i.e., \( a_{1i}, a_{2i} \ldots a_{pi} \) are the elements of the eigenvector corresponding to the \( l \)th eigenvalue of the correlation matrix of the response variables.

**Step 3.** Since a larger PCS is always desired, normalized the principal component scores by using Equation:

\[ X_{il} = \frac{PCS_{il} - PCS_{il}^{\min}}{PCS_{il}^{\max} - PCS_{il}^{\min}} \]  

**Step 4.** Based on normalised principal component scores, calculate the grey relational coefficient (\( \gamma_{il} \)) computed as follows:

\[ \gamma_{il} = \frac{\Delta_{ml} + \xi \Delta_{m\max}}{\Delta_{ml} + \xi \Delta_{m\max}} \]  

Where, \( \Delta_{ml} = |1 - X_{il}| \), \( \xi \) is the distinguishing coefficient which range varies from [0, 1].

**Step 5.** The OQPI’s values can be obtained using the following expression:

\[ OQPI_l = \sum_{i=1}^{n} w_i Y_{il} \]  

Where, \( \sum_{i=1}^{n} w_i = 1 \)

**Step 6.** Use geometric average to calculate the factor effects based on OQPI values, and then decide the optimal factor-level combination based on the higher-the-better factor effects.

The computed OQPI values corresponding to different trials for the PCA-based method are shown in Table 3 which gives the level averages of the control factors and the optimal machining conditions separately for this three responses are calculated by reduction of principal components which shown in Figure 1 and the values are given in Table 5. A confirmatory test is conducted with the optimal parameter setting and shown in Table 5.
TABLE 1. Input parameters with their unit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Level</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge current</td>
<td>Ip</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Pulse on Time</td>
<td>T_{on}</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Lift Time</td>
<td>T_{up}</td>
<td>0.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Work time</td>
<td>T_w</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Inter electrode gap</td>
<td>EIG</td>
<td>90</td>
<td>70</td>
</tr>
</tbody>
</table>

Fixed Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Level</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty Cycle</td>
<td>(ζ)</td>
<td>90</td>
<td>%</td>
</tr>
<tr>
<td>Voltage</td>
<td>V</td>
<td>45</td>
<td>V</td>
</tr>
<tr>
<td>Flasing Pressure</td>
<td>Fp</td>
<td>0.3</td>
<td>Kgf/cm²</td>
</tr>
</tbody>
</table>

Figure 1 For OQPI

TABLE 2. Eigenvalues and eigenvectors from original s/n ratios

<table>
<thead>
<tr>
<th>Principal Component</th>
<th>Eigen Value</th>
<th>Proportion of variation explained</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>2.1798</td>
<td>0.727</td>
<td>[0.541, -0.580, 0.609]</td>
</tr>
<tr>
<td>Second</td>
<td>0.5291</td>
<td>0.176</td>
<td>[0.808, 0.558, -0.187]</td>
</tr>
<tr>
<td>Third</td>
<td>0.2911</td>
<td>0.097</td>
<td>[0.231, -0.594, -0.771]</td>
</tr>
</tbody>
</table>

TABLE 3. Mean values of OQPI

<table>
<thead>
<tr>
<th>Level</th>
<th>Ip</th>
<th>Ton</th>
<th>Tup</th>
<th>Tw</th>
<th>IEG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5808</td>
<td>0.4289</td>
<td>0.4538</td>
<td>0.4952</td>
<td>0.4876</td>
</tr>
<tr>
<td>2</td>
<td>0.5010</td>
<td>0.4955</td>
<td>0.4871</td>
<td>0.4835</td>
<td>0.5127</td>
</tr>
<tr>
<td>3</td>
<td>0.4031</td>
<td>0.5604</td>
<td>0.5439</td>
<td>0.5061</td>
<td>0.4845</td>
</tr>
</tbody>
</table>

Conclusion

Grey relational analysis is a very useful tool for predicting optimization of the multi response problems and it does not involve complicated mathematical theory or computation. So, it can be employed by the engineers without a strong statistical background. The proposed procedure has been shown to be as successful in obtaining the optimal parameter conditions i.e., Ip1 Ton3 Tup3 Tw3 IEG2 based on the higher-the-better factor effects using Grey Relational Analysis (GRA) employing principal component analysis is most able to consistently obtain the optimal parameter design for correlated multiple quality characteristics.

References
