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## Vibration and Buckling of Composite Plates in Hygrothermal Environment

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### ABSTRACT

The present study deals with numerical investigation on vibration and buckling behavior of laminated composite plates subjected to varying temperature and moisture concentration. Quantitative results are presented to show the effects of geometry, material and lamination parameters of woven fiber laminate on vibration and buckling of composite plates for different temperature and moisture concentrations.

### 1. Introduction

Composite materials are mainly used in aerospace, naval and high performance civil engineering structures. The structures are often exposed to high temperature and moisture. As a result, it changes their deformation, vibration, static and dynamic stability characteristics. Chen and Chen [1] studied the free vibration of the laminated rectangular composite plate exposed to steady state hygrothermal environment. Sai Ram and Sinha [2] investigated the effects of moisture and temperature on the free vibration of laminated composite plates using finite element method. The analysis also accounts for lamina material properties at elevated moisture concentration and temperature. Chen and Chou [3] examined the free vibration analysis of orthogonal-woven fabric composites. The fabrics are composed of two sets of mutually orthogonal yarns of either the same materials or different materials. Naik *et al.* [4] investigated the damage in woven-fabric composites subjected to low-velocity impact. The behavior of woven fabric laminated composite plates has been studied under transverse central low-velocity point impact by using a modified Hertz law and a 3D transient finite-

element analysis code. Babu and Kant [5] proposed with a refined higher order finite element models for thermal buckling of laminated composite plates. Patel *et al.* [6] studied the hygrothermal effects on the structural behavior of thick composite laminates using higher-order theory. The analysis is carried out employing a C<sup>0</sup> QUAD-8 isoparametric higher-order finite element. Zhen and Wanji [7] proposed with a mathematical model for determination of the buckling analysis of angle-ply composite and sandwich plates. Analysis is based on the global-local higher order theory with combination of geometric stiffness matrix. Singh and Verma [8] investigated the hygrothermal effects on the buckling of laminated composite plates with random geometric and material properties. A C<sup>0</sup> finite element model based on higher order shear deformation theory is used for deriving the eigenvalue problem. Pandey *et al.* [9] studied the hygrothermoelastic post buckling response of laminated composite plates. The quadratic extrapolation technique and first converging finite double chebyshev series are used for linearization and spatial discretization of the governing non linear equations of equilibrium

### 2. Mathematical Formulation

The mathematical formulation for buckling effects of laminated composite plates subjected to moisture and temperature are presented. Consider a laminated plate of uniform thickness 't' consisting of a number of thin laminae, each

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of which may be arbitrarily oriented at an angle 'θ' with reference to the X-axis of the co-ordinate system as shown in Figures 1.

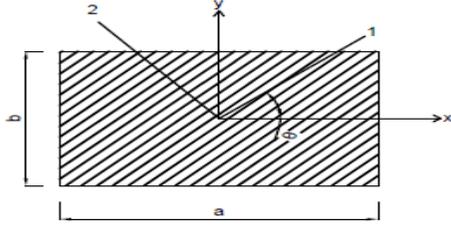


Figure 1. Arbitrarily oriented laminated plate  
Governing Equations

The governing equations for the structural behavior of the laminated composite plates are derived on the basis of first order

$$\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$$

$$\{F^N\} = \{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, 0, 0\}^T$$

$$\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \gamma_{xy}, K_x, K_y, K_{xy}, \phi_x, \phi_y\}^T$$

shear deformation theory. The constitutive relations for the plate subjected to moisture and temperature are given by:

$$\{F\} = [D]\{\varepsilon\} - \{F^N\}$$

(1) Where

Where,  $N_x, N_y, N_{xy}$  = in-plane internal stress resultants.

$M_x, M_y, M_{xy}$  = internal moment resultants.

$Q_x, Q_y$  = transverse shear resultants.

$N_x^N, N_y^N, N_{xy}^N$  = in-plane non-mechanical stress resultants due to moisture and temperature.

$M_x^N, M_y^N, M_{xy}^N$  = non-mechanical moment resultants due to moisture and temperature.

$\varepsilon_x, \varepsilon_y, \gamma_{xy}$  = in-plane strains of the mid-plane.

$K_x, K_y, K_{xy}$  = Curvature of the plate

$\phi_x, \phi_y$  = Shear rotations in xz and yz planes, respectively.

Derivation of Element matrices

The element matrices are derived as given below:

Element stiffness matrix

$$[K_e] = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B] J |d\xi d\eta$$

(4)

Element initial stress stiffness matrix (due to non mechanical loads)

$$[K_g] = \int_{-1}^{+1} \int_{-1}^{+1} [G]^T [S][G] J |d\xi d\eta$$

(5)

Element mass matrix

$$[M] = \int_{-1}^{+1} \int_{-1}^{+1} [N]^T [P][N] J |d\xi d\eta$$

(6)

The element load vector due to hygrothermal forces and moments is given by

$$\{P_e^N\} = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T \{F^N\} J |d\xi d\eta$$

(7)

### 3. Results and Discussions

In the present investigation, results are presented for laminates subjected to uniform distribution of temperature and moisture concentration. Sixteen layered Glass fiber/epoxy laminates with clamped-free-clamped free boundary condition have been analyzed experimentally and all computations are made with FEM in MATLAB code. The vibration frequencies in Hz are reported. The aspect ratios considered are 0.5, 1.0 and 2 as shown in figure 2 and 3. As increase in aspect ratios the frequencies of vibration decreases with increase in temperature and moisture concentration due to reduction of stiffness of the plate. It is observed that for aspect ratio 1 and 2 beyond temperature 400K and moisture concentration 0.75%, frequency of vibration is decreased and approaches to zero with increase in temperature and moisture concentration due to Hygrothermal buckling starts beyond that point.

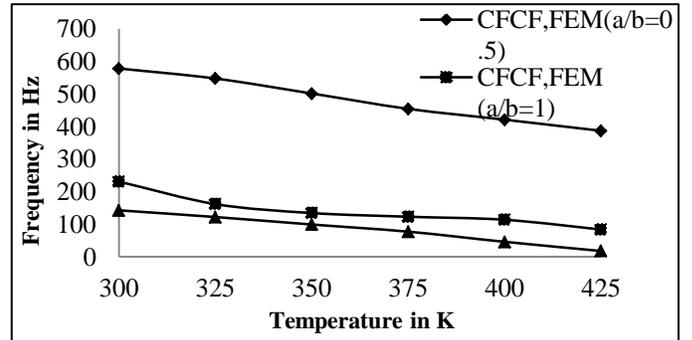


Figure 2. Variation of frequency in Hz with temperature for (c-f-c-f) of 16 layers [0/0]<sub>4s</sub> composite plates.

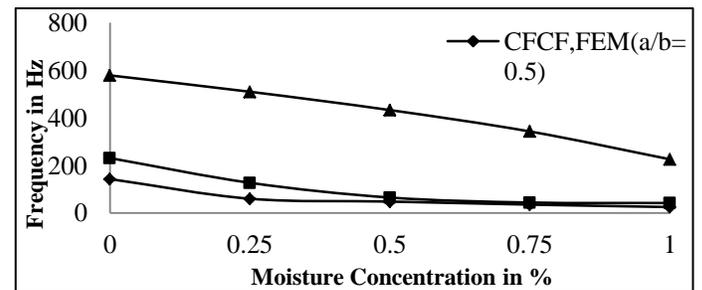


Figure 3. Variation of frequency in Hz with moisture concentration for simply supported (C-F-C-F) of [0/0]<sub>4s</sub> composite plates.

It is observed that the critical buckling load in KN is highest for  $a/b=0.5$  and stability decreases with increase in aspect ratios of laminated composite plates in hygrothermal environment. For higher buckling load strength corresponding to  $a/b=1$ , and 2 is almost same with rise in temperature and moisture concentration. Hygrothermal buckling also starts at room temperature about 300K which is known as stress-free level. Similarly hygroscopic buckling will start at 0.25% of moisture concentration which is also known as stress-free level. As the hygrothermal stress resultants are integrated quantities, their effect increases with the absorption of more moisture until equilibrium is reached.

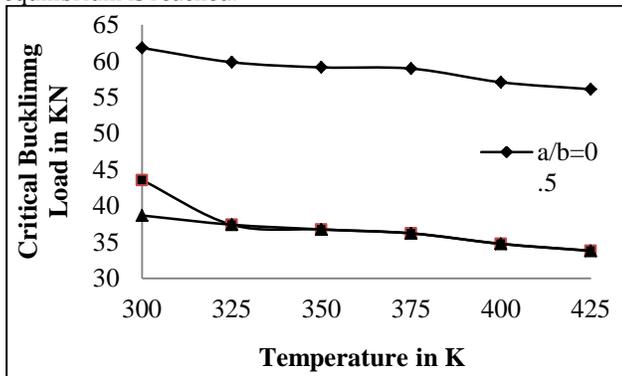


Figure 4. Variation of critical buckling load in KN with temperature of 16 layers  $(0/90)_{8S}$  composite plates (C-F-C-F).

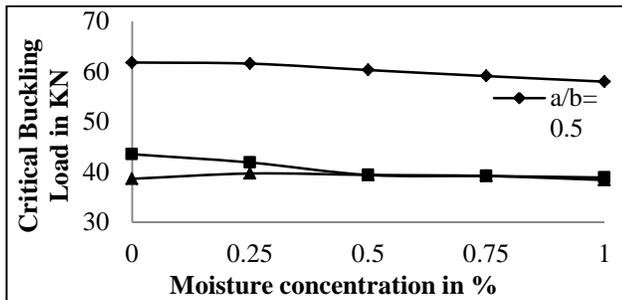


Figure 5. Variation of critical buckling load in KN with moisture concentration of 16 layers  $(0/90)_{8S}$  composite plates (C-F-C-F).

#### 4. Conclusion

- There is a good agreement between the experimental and numerical results for natural frequencies and buckling of laminated composite plates at different temperature and moisture.
- As increase in aspect ratios the frequency of vibration decreased in hygrothermal environment
- The critical buckling load decreased with increase in aspect ratio

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