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Stress Intensity Factors for Single Cracked Pressurized Cylinder

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A B S T R A C T

The stress intensity factor is the linear elastic fracture mechanics parameter that relates remote load, crack size and structural geometry. It predicts very accurately the stress state. In this work SIF (Stress Intensity Factor) for a single cracked pressurized cylinder is calculated. Ansys10 Finite Element Analysis software was used in this study. This software was used to obtain the nodal forces at the crack tip as well as the displacements in the vicinity of the crack tip. These two parameters were then used in the modified virtual crack closure technique to obtain the stress intensity factors. Very good agreement between the finite element stresses and the theoretical stresses is seen.

Introduction

A thick-walled cylinder is a pressure vessel whose wall thickness is greater than one tenth of its inner radius [1]. Thick-walled cylinders are used for a variety of applications such as industrial compressed air receivers, domestic hot water storage tanks, diving cylinders, recompression chambers, distillation towers, autoclaves and many other vessels in mining or oil refineries and petrochemical plants. They are also used as nuclear reactor vessels, space ships and submarines, pneumatic and hydraulic reservoirs under pressure, brake reservoirs and storage vessels of liquefied gases such as ammonia, chlorine, propane, butane and LPG [2].

In a thick-walled pressure vessel, sites for crack initiation are either caused by design features such as notches, or by accident, e.g., flaws due to manufacturing process or handling. Cracks may also be formed in pressure vessels subjected to extreme loading conditions [3]. The length of a crack in a pressure vessel may increase due to the application of repeated loads or due to a combination of loads and environmental attack. As its length increases, so does the stress concentration induced by it [4]. Due to the presence of the crack, the residual strength of the pressure vessel decreases progressively with increasing crack size until it becomes so low that the pressure vessel cannot withstand high pressures. Even if the cylinder is not subjected to very high pressures, the crack will continue to grow until the residual strength becomes so low that fracture occurs under normal service pressure [4].

By 1992, the stress intensity factors data available consisted of more than 400 values of KIP (the K due to internal pressure, for different crack arrays and for a

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wide range of crack lengths), as well as more than 200 values of KIA (the negative K due to the compressive residual stress field induced by the auto-frettage process for different configurations of crack arrays and various levels of auto-frettage) [5]. A detailed analysis of the available data revealed that the documented results could be classified into two categories namely `sparse crack arrays' and `dense crack arrays'. For sparse crack arrays both KIP and KIA were directly proportional to the crack length, `a', but for the dense case, they depended primarily on the inter-crack spacing, `d', and were practically crack length independent. Furthermore, the inter-crack aspect ratio, a/d, was found to be the sole parameter that determined which of the two categories each case belonged to.

Based on these results, and using least squares approach, stress intensity factor approximate formulae for KIP and KIA for sparse and dense crack arrays were developed by Perl [5]. The expressions were simple but of very good engineering accuracy and are applicable to the whole gamut of existing results. Cylinders with up to 1024 radial cracks were considered.

The problem of determining stress intensity factors in pressure vessels has been addressed by a number of researchers [6-7]. Raju and Newman [8] revisited stress-intensity factor influence coefficients for a wide range of semi-elliptical surface cracks on the inside or outside of a cylinder. Googarchin and Ghajar [9] derived the General point load weight function (GPLWF) for a longitudinal semi-elliptical crack in a thick-walled cylinder with outer radius to inner radius ratio of 1.25. Carpinteri et al. [10] analyzed the influence of notches on the fatigue life of metallic structural components, particularly, notches with different shapes were examined in the case of both double-curvature shells and round bars under mode I loading.

Most of this research has been done using methods such as the compounding method, force method, displacement extrapolation, J-integral and singularity subtraction technique. The virtual crack closure technique and its modified version (MVCCT) have also been used in determining SIFs but mostly in cracked straight edged components [11]. It is known that edge cracks are more dangerous than interior cracks [12] because the presence of defects significantly affects the fatigue life [13]. The authors have used the MVCCT has for determining stress intensity factors in thick-walled cylinders with multiple cracks[14].

2. Background Stresses in thick-walled cylinders

Stresses in thick-walled cylinders are classified according to the directions in which they act. There are three classes of stresses which are found in a loaded cylinder and these are;

Hoop or tangential stress, σ_θ

Radial stress, σ_r

Longitudinal/axial stress, σ_l

Any of these stresses at a particular point(R) in a cylinder can be calculated using the Lame's equation. i.e.

$$\sigma_\theta = A_i + \frac{A_2}{R^2} \quad (1)$$

$$\sigma_r = A_1 - \frac{A_2}{R^2} \quad (2)$$

$$\sigma_l = A_1$$

where,

$$A_1 = \frac{P_i R_i^2 - P_o R_o^2}{R_o^2 - R_i^2}$$

$$A_2 = \frac{(P_i - P_o) R_o^2 R_i^2}{R_o^2 - R_i^2}$$

P_i is the internal pressure acting on the cylinder

P_o is the external pressure acting on the cylinder

R_i is the inner radius of the cylinder

R_o is the outer radius of the cylinder

Stress concentration factor

The stress concentration factor is the ratio of the greatest stress at the crack tip to the corresponding nominal stress that existed at the location of the crack tip before the formation of the crack. That is;

$$K_f = \frac{\sigma_{max}}{\sigma_o}$$

The severity of the stress concentration depends on the geometry of the crack. Designers should therefore always try to reduce stress concentrations as much as possible in order to avoid fatigue problems.

Stress intensity factor

The stress intensity factor is the linear elastic fracture mechanics parameter that relates remote load, crack size and structural geometry. It predicts very accurately the stress state (`stress intensity') near the tip of a crack caused by a remote load or residual stress. The crack initiation life is highly dependent on the stress concentration factor K_f value. The crack initiation period is followed by the fatigue crack growth period. For a crack, the K_f value is no longer a meaningful concept to indicate the severity of the stress distribution around the crack tip [15].

The difference between a notch and a crack can be illustrated by considering an elliptical hole. In an infinite sheet loaded in tension, K_f is given by [15]

$$k_f = 1 + 2\sqrt{\frac{a_1}{\rho}}$$

$$\rho = \frac{b_1^2}{a_1}$$

where;
a_1 is the major semi-axis of the elliptical hole
b_1 is the minor semi-axis of the elliptical hole
ρ is the tip radius of the elliptical hole
Because a crack is a notch with zero tip radius, K_f would become infinite, and this would be true for any crack length. The stress intensity factor is therefore a better concept of describing the severity of the stress distribution around the crack tip. K can be expressed as;

$$K = \lambda\rho\sqrt{\pi a}$$

where σ is the remote loading stress, `a' is the crack length and λ is a dimensionless factor which depends on the geometry of the specimen or structural component. The important feature therefore is that the stress intensity factor fully determines the stress field around the crack tip.

The stress intensity factor is used to determine the fracture toughness of most materials. Fracture toughness is an indication of the amount of stress required to propagate a pre-existing flaw. It is a very important material property since the occurrence of flaws is not completely avoidable in the processing, fabrication or service of a material/component. Flaws may appear as cracks, voids, metallurgical inclusions, weld defects, design discontinuities or some combination thereof. Since engineers can never be totally sure that a material is flawless, it is common practice to assume that a flaw of chosen size will be present in some number of components and use the linear elastic fracture mechanics (LEFM) approach to design critical components. This approach uses the flaw size and features, component geometry, loading conditions and the material property called fracture toughness to evaluate the ability of a component containing flaw to resist fracture.

3. Methodology

The Ansys10 Finite Element Analysis software was used in this study. This software was used to obtain the nodal forces at the crack tip as well as the displacements in the

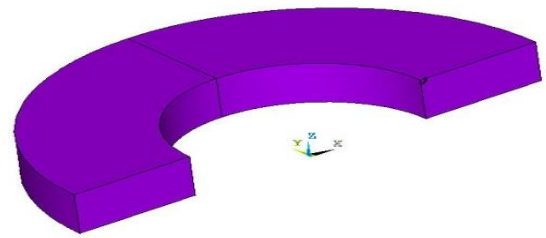


Fig. 1. 3D single crack Model

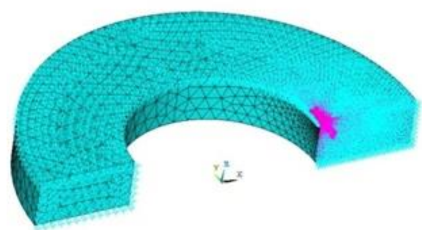


Fig. 2. Refined mesh at the crack tip

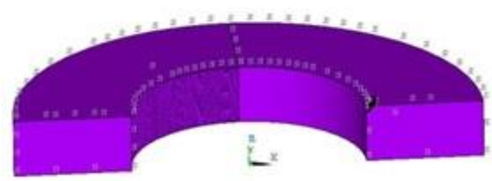
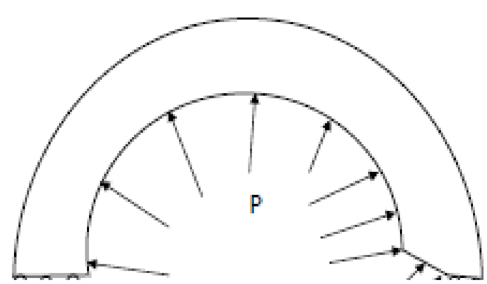


Fig. 3. Model with boundary conditions



vicinity of the crack tip. These two parameters were then used in the modified virtual crack closure technique to obtain the stress intensity factors. Cylinders with single cracks as well as those with multiple cracks were considered. The following parameters were varied during the analysis of the cylinders:

The number of cracks. (The variation in number of cracks ultimately led to a variation in the inter-crack spacing).

The crack length to cylinder thickness ratio (a/t).

The diameter ratio of the cylinders.

For cylinders with a single crack, the analysis was done by considering one half of the cylinder so as to take advantage of symmetry. Figure 1 shows the model that was used but the crack size shown has been exaggerated for the sake of clarity. The model structural properties were chosen as E, (Young's Modulus of Elasticity) =210 GPa and ν, (Poisson's ratio) =0.3. It was then meshed using solid-Tet-10node-187 elements. These elements were chosen because they could be arranged in a regular manner around the crack tip and also because they allowed for sufficient mesh refinement around the crack tip.

During the meshing, the model was first freely meshed. The meshing of the whole model was then refined. Finally mesh refinement was done at the crack tip and this is illustrated in Figure 2.

The next stage of the analysis involved the application of boundary conditions. The displacement boundary conditions were applied to prevent the cylinder from rotating about its own axis. The areas in the model were also numbered so as to ensure that the boundary conditions were applied in their appropriate areas. Figure 3 shows the model with displacement boundary conditions on area A4 and A7 whereas Figure 4 shows the front view of the same model. Internal pressure was applied on the inner surface of the cylinder as well as on the crack face while making sure that yielding did not occur at any point in the cylinder. The precondition conjugate solver was chosen to carry out the final analysis since it is fast and accurate. Figure 5 shows the stress distribution along the crack tip. It is clear from this figure that the stress along the crack tip is higher than the stress at any other point on the cylinder.

After the solution, the deformed and the unreformed model were observed. The maximum stress was also checked so as to verify that it was below the model yield stress. The variation of the radial, hoop and axial stresses was also examined along the crack, i.e. the variation of the stresses from the point of crack initiation on the inner surface of the cylinder up to the crack tip.

The node numbering of the whole model was activated and then a section of the crack tip was selected to be used in the analysis. For the selected portion, nodal forces in the hoop/circumferential direction were obtained at five consecutive nodes located along the crack tip. The nodal displacements in the hoop/circumferential direction were also obtained at other five consecutive nodes near the crack tip.

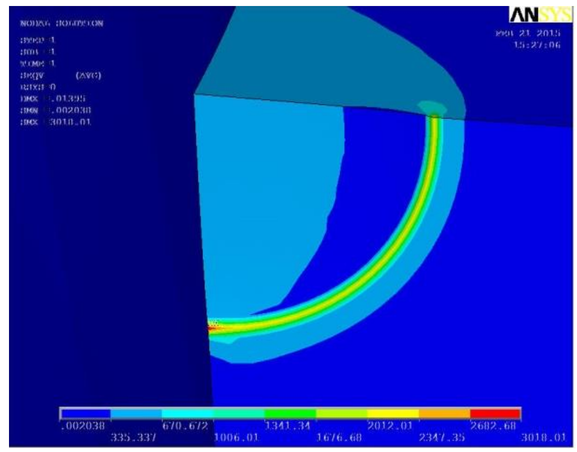


Fig. 5. Stresses at the crack tip

4. Results and Discussion

Model validation

The radial, hoop and axial stresses obtained from the finite element model were compared with their corresponding theoretical stresses obtained using the Lame's equations, i.e. equations 1 and 2. Figure 6 shows the comparison of these stresses for a flawless cylinder with a thickness ratio of Y=1.5.

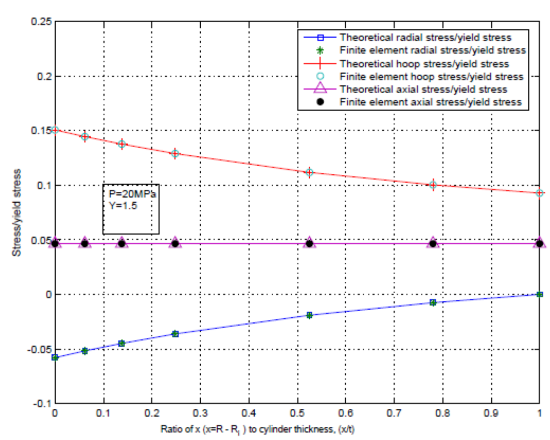


Fig. 6. Theoretical versus finite element stresses

From the graph, it was evident that there was very good agreement between the finite element stresses and the theoretical stresses. Both the hoop stress and the radial stress (which has a negative value since it is compressive in nature) were found to decrease with increasing radius of the cylinder. This was because the cylinder considered was subjected to internal pressure only. The axial stresses were found to have a constant value throughout the cylinder thickness and this was the pressure required to counteract the effect of the internal pressure on the cylinder ends. Since there was a good agreement between the flawless cylinder's finite element stresses and theoretical stresses, it was now possible to confidently use the Ansys10 Finite Element Analysis Software to analyze models of cracked thick-walled cylinders.

Figure 7 shows the comparison of the finite element stresses with those obtained using the Lame's equations for a cylinder with a thickness ratio of 1.5 having a crack whose length extended up to half the thickness of the cylinder. From the figure, it is clear that the presence of the crack introduced a region of stress concentration at the crack tip where the finite element stresses were very high. The hoop stress at the crack tip was found to be higher than the other stresses at the crack tip and this was due to the fact that the pressure loading on both crack faces was acting in the circumferential direction. This also proved that the cylinder under consideration was more likely to fail in tensile mode and thus showed why the mode I stress intensity factor is of greater interest than the other stress intensity factors.

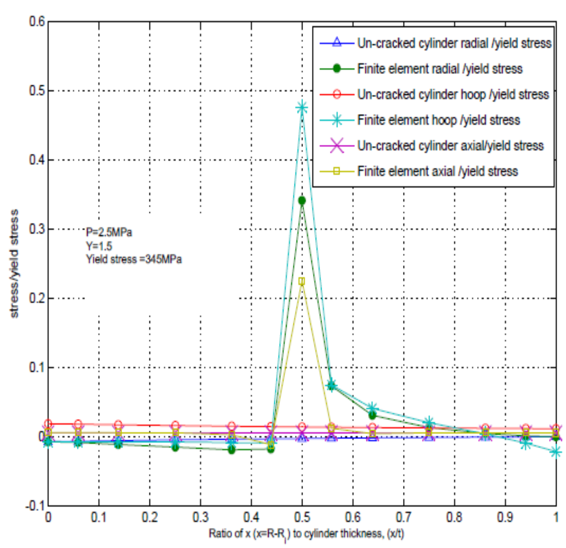


Fig. 7. A cracked cylinder finite element stresses versus an un-cracked cylinder theoretical stresses

Stress intensity factors for cylinders with a single crack

Stress intensity factors for cylinders with a single crack were obtained using the modified virtual crack closure technique and compared with those already in literature [15]. This was done for cylinders with Y=1.5, Y=2.0 and Y=2.5. There was good agreement with the published results for 0.1_ a/t_0.7. Figures 8 to 10 shows the comparison of the results obtained. Km refers to the results obtained using the modified virtual crack closure technique whereas K_k refers to those obtained from literature [15]. The stress intensity factor was found to increase as the length of the cracks were increased. This was because, as the crack length increased, the area of the crack face under pressure increased. Since force is a product of both area and pressure, the increased area resulted in increased circumferential force at the crack tip. The stress intensity factor was found to decrease with increasing diameter ratio of the cylinder. This was because, cylinders with a large diameter ratio are stronger than those with a small diameter ratio.

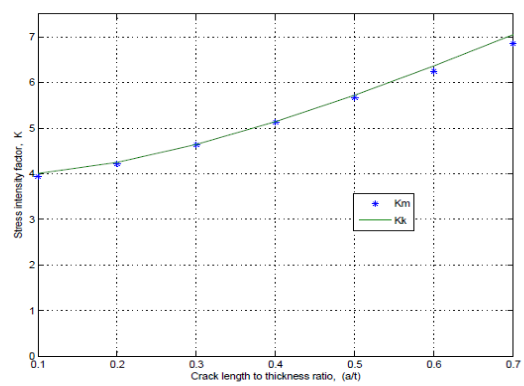


Fig. 8. Comparison of Km with K_k for Y=1.5

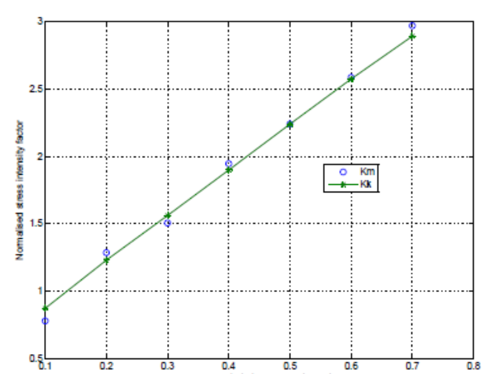


Fig. 9. Comparison of Km with K_k for Y=2

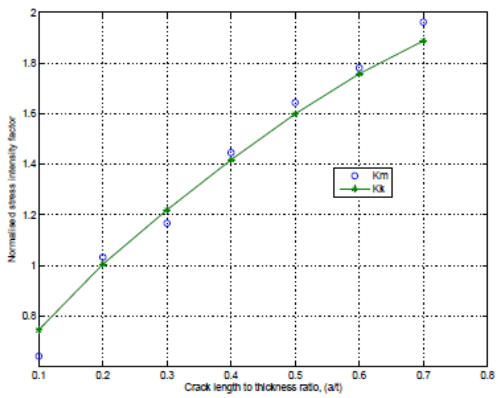


Fig. 10. Comparison of Km with K_k for Y=2.0

5. Conclusions

The intend of this work was to use the modified virtual crack closure technique to find the stress intensity factors in multiply cracked thick walled cylinders. The Ansys14 Finite Element Analysis software was used to get the nodal forces at the crack tip and the nodal displacements near the crack tip. These parameters enabled the purpose of K values for cylinders with different diameter ratios, crack length to thickness ratio and number of cracks. The solid-Tet-10node-187 elements were used in meshing the models and this was due to their regular agreement around the crack tip that enabled the nodal displacements to be obtained at a constant distance from the crack tip. The models utilized were validated by comparing the finite element hoop, radial and axial stresses with their equivalent values which were obtained using Lame's equations.

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