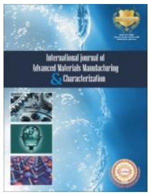

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Non-dimensional Analysis of Orthotropic Plate with Circular Cutout

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A R T I C L E I N F O

Article history:
Received: 09-08-2015
Accepted: 12-09-2015

Keywords:
SCF,
CircularCutout,
Non-dimensional Parameters,
Polar angle,
Orthotropic.

A B S T R A C T

Stress concentration is the localization of stresses due to discontinuation or abruption in the geometry. To find the localized stresses, Stress Concentration Factor (SCF) is used. SCF depends on the material and geometry of the specimen. In the present paper an orthotropic plate with a central circular cutout subjected to normal pressure is studied. Based on material properties variation of the non-dimensional parameters E_2/E_1, G_{12}/E_1 and μ_{12} have been considered within their feasible range. The effect of these non-dimensional parameters at different values of polar angle on SCF is studied here. Formulation of mathematical equation for SCF has been done in MATLAB and subsequently results are presented here. The work successfully highlights that for orthotropic material (unlike isotropic materials) SCF depends on material properties. It increases with increase in E_2/E_1 ratio and decreases with increase in μ_{12} and G_{12}/E_1 ratio.

Introduction

It has been proved by several experiments that the stress concentration in vicinity of the cutout in perforated plates is the limiting factor and this actually decides the condition of the structure subjected to loading. Precise knowledge of SCF of orthotropic plate is required as they are often used in critical conditions. Stress distribution and deflection of the orthotropic plates has been the area of interest of many design and structural engineers.

Muskhelischwili [1] and Lekhnitskii [2] were among the first to suggest the analytical approach for the study of stress concentration.This had been adapted as the fundamental work to develop solutions for stress concentration by many other researchers[3-5]. Deb Nath et al. [6] used the finite-difference technique based on the displacement potential approach and extended it to solve elastic plane problems of orthotropic composite materials with various geometric disruptions likeholes, arbitrary defects, notches etc. Ashrafi et al. [7] presented a 3D graded boundary element model adequate for studying elastostatic response of perforated heterogeneous plates fabricated from functionally graded materials (FGMs). The analytical investigation on the effect of stacking sequence, ply groups, loading angles, materials and corner radii on the failure strength and moment distribution in symmetric laminated composite plate weakened by a square hole, was given by Patel and Sharma [8]. They have also provided general solution for moment distribution around a circular/elliptical/triangular shaped hole in a laminated

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- Doi: http://dx.doi.org/10.11127/ijammc.2015.09.03

composite plate subjected to a bending/twisting moment [9]. Using Muskhelishvili's complex variable method, Sharma [10] determined the stress concentration around circular, elliptical and triangular cutouts in laminated composite infinite plate subjected to arbitrary biaxial loading at infinity. Jain et al. [11] analyzed the effect of different fibre orientation upon stress concentration factor around the hole. Dependence of the deflection and stress concentration factor on cutout geometry and size was observed by Kalita and Haldar [12]. Yeh et al. [13] presented the effect of various offaxis angles and lamina material properties on the Poisson's ratio and the stress distribution of a circular cutout in an orthotropic composite plate. Kalita et al. [14-16] has presented several works on effect of cutout orientation on stress and their mitigation. The effect of concentric circular cutout hole on multi-layer of epoxy composite laminated plates has been presented by many researchers[17].The relationship between shear modulus G, bulk modulus K, and Poisson's ratio for isotropic materials is $G=(3K(1-2\mu))/(2(1+\mu))$. Poisson's ratio of a solid is defined as the lateral contraction strain divided by the longitudinal extension strain in a simple tension experiment. In almost all materials the Poisson's ratio, usually denoted by μ, is positive. Most common materials have a Poisson's ratio close to 1/3, however rubbery materials have values approaching 1/2; they readily undergo shear deformations but resist volumetric (bulk) deformation, so G << K in the above given relation.A number of researchershave shown that materials with negative Poisson's ratio can be synthesized [18-22]. Some such materials have already found applicability in vibration dampers, crash helmets, body armor etc. A solid with μ ~ -1 would be the opposite of rubber: difficult to shear but easy to deform volumetrically: G >> K.

The brief literature survey highlights that though a lot of work has been done in the field of stress concentration, there still remain a lot of undiscovered facts. Here we have considered a perforated plate with a central circular cutout loaded under normal pressure distributed uniformly along the opening edge. We have studied the influence of various material properties like E_1, E_2, G_{12}, Poisson's ratio etc. by non-dimensional analysis using MATLAB. We have considered hydrostatic tension as the loading here and the analysis has been carried out for negative Poisson's ratio as well.

Problem Definition

With the help of complex variable method stress-strain distribution is derived for an anisotropic infinite plate having a cut out throughout its thickness. If an equal tensile stress in two mutually perpendicular direction is applied over the orthotropic plate, the normal stress component for a circular opening is

$$\sigma_\theta = P\frac{E_\theta}{E_1}(-k + n(\sin^2\theta + k\cos^2\theta) + (1 + \mu_1{}^2)(1+\mu_2{}^2).\sin^2\theta.\cos^2\theta)$$

Where $k = \sqrt{\frac{E_1}{E_2}}$

$$n = \sqrt{\frac{E_1}{G_{12}} - 2\vartheta_{12} + 2\sqrt{k}}$$

$$\mu_1.\mu_2 = -k$$

$$\mu_1{}^2 + \mu_2{}^2 = 2\vartheta_{12} - \frac{E_1}{G_{12}}$$

The θ is the polar angle measured form the x-axis, whereas E_θ is the young's modulus along the polar angle, whose relation with other elastic constant is given by,

$$\frac{1}{E_\theta} = \frac{\sin^4\theta}{E_1} + \left(\frac{1}{G_{12}} - \frac{2\mu_{12}}{E_1}\right)\sin^2\theta.\cos^2\theta + \frac{\cos^4\theta}{E_2}$$

For an Isotropic plate, $\sigma_\theta = P$

$$SCF = \frac{\sigma_\theta}{\sigma_{nominal}}$$

Here an orthotropic plate containing a circular cut out subjected to normal pressure distributed uniformly along the opening edge is considered.

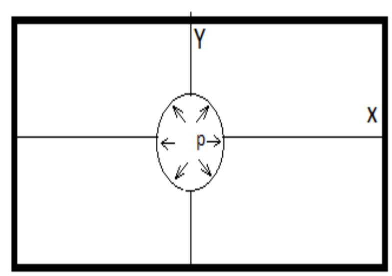

Fig. 1. Perforated orthotropic plate under normal pressure distributed uniformly along the opening edge.

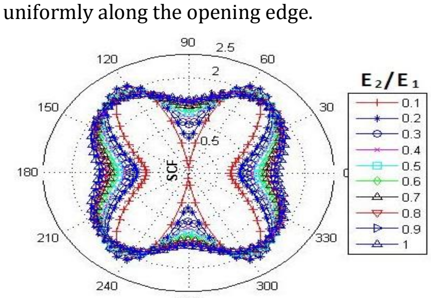

Fig. 2. SCF V/s θ for various E_2/E_1at P=1Mpa, E_1=44.7 GPa, G_{12}/E_1=0.2 and μ_{12}= 0.3.

The three materials constant E_1, E_2 and G_{12} of the plate have been consider as a variable parameter for the analysis. Three non-dimensional ratios E_2/E_1, G_{12}/E_1 and μ_{12} have been considered based on above material constant. Various cases of combination of these parameters have been considered in this study. Ranges of values for $E_2/E_1$and $G_{12}/E_1$is 0.1 to 1.0 and for μ_{12} is 0.1 to 0.5. First Maximum stresses are determined and then SCF are calculated. Here the applied stress is considered as the nominal stress.

Results and Discussion

The SCF for an orthotropic plate is symmetrical with respect to the opening centre of the circular cutout. Hence variation of SCF is considered only in the range of $0^o \leq \theta \leq 180^o$ for an orthotropic plate. The non-dimensional ratios have been varied within the range and the polar angle θ up to 180^o and symmetry has been followed.

Fig. 2 shows SCF distribution for various equidistant values of non-dimensional parameter E_2/E_1 in range of polar angle $0^o \leq \theta \leq 360^o$ keeping values of other non-dimensional parameter as a constant. The material properties considered here are E_1= 44.7 GPa, G_{12}/E_1=0.2 and μ_{12}= 0.3. 1 GPa normal pressure load distributed uniformly over the edge of circular opening at the centre of the orthotropic plate as shown in Figure 1 is considered. For E_2/E_1=0.1 it is seen that SCF increases with increase in polar angle up to 60^o after which it starts decreasing and attains a minimum value at 90^o. At E_2/E_1=0.2 and 0.3 SCF increases with increase in polar angle from 0^o up to 55^o, where it is maximum. For E_2/E_1=0.4 to 0.9 the SCF increases with increase in polar angle from 0^o to 50^o and then starts decreasing till 90^o. For E_2/E_1=0.4 to 0.9 the SCF is minimum at θ=0^o. At E_2/E_1=1 the SCF increases from 0^o to 45^o and then decreases until 90^o. Since there is no variation in E_1andE_2 it is obvious that the variation in SCF will be up to 45^o and from there on it would be symmetrical for every 45^o alternately increasing and then decreasing SCF. Hence for E_2/E_1=1 minimum value is seen at 0^o, 90^o, 135^o, 180^o and so on. At θ=0^o the SCF increases almost linearly with increase in E_2/E_1. At θ=45^o SCF increases from E_2/E_1=0.1 to 0.3 and after that decreases with increase in E_2/E_1. At θ=90^o SCF increases steeply with E_2/E_1.

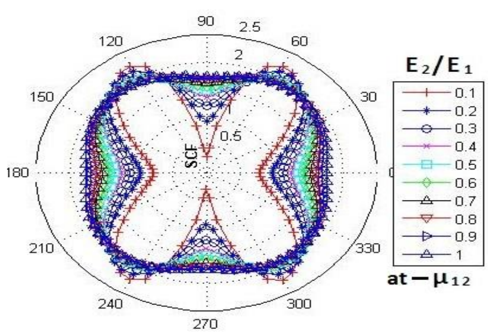

Fig. 3. SCF V/s θ for various E_2/E_1at P=1Mpa, E_1=44.7 GPa, G_{12}/E_1=0.2 and μ_{12}= - 0.3.

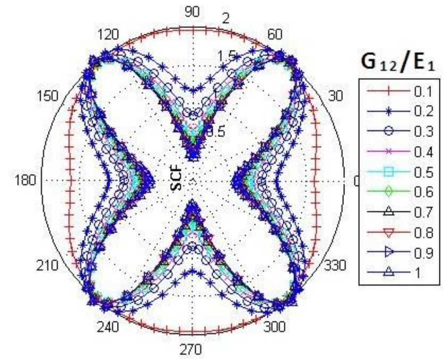

Fig. 4. SCF V/s θ for variousG_{12}/E_1at P=1Mpa, E_1=44.7 GPa, E_2/E_1=0.4 and μ_{12}=0.3.

Fig. 3 shows SCF for various equidistant values of non-dimensional parameter E_2/E_1 in range of polar angle $0^o \leq \theta \leq 360^o$ keeping all parameters as for Fig. 2 but a negative Poisson's ratio. For E_2/E_1=0.1 and 0.2 SCF increases with increase in polar angle from 0^o up to 60^o, where it is maximum and is minimum at 90^o. For E_2/E_1=0.3 to 0.4 the SCF increases with increase in polar angle from 0^o to 55^o and then starts decreasing till 90^o but is minimum at initial polar angle θ=0^o. For E_2/E_1=0.5 to 0.9 the SCF is minimum at θ=0^o and is maximum at θ=50^o. At E_2/E_1=1 the SCF increases from 0^o to 45^o and then decreases until 90^o. Since there is no variation in E_1andE_2 it is obvious that the variation in SCF will be up to 45^o and from there on it would be symmetrical for every 45^o alternately increasing and then decreasing SCF. Hence for E_2/E_1=1 minimum value is seen at 0^o, 90^o, 135^o, 180^o and so on.

Fig. 4 shows variation of SCF with change in G_{12}/E_1. Other non-dimensional parameters considered here are P=1Gpa, E_1=44.7 GPa, E_2/E_1=0.4 and μ_{12}= 0.3. For G_{12}/E_1=0.1 SCF increases with increase in polar angle up to 70^o after which it starts decreasing up to 90^o. At G_{12}/E_1=0.1 and 0.2 SCF is minimum at 0^o and for G_{12}/E_1=0.3 to 1 SCF is minimum at $90^o$and maximum at 50^o. At θ=0^o and θ=90^o the SCF decreases monotonically with increase in G_{12}/E_1. At θ=45^o variation of SCF with increases in G_{12}/E_1 is negligible.

Fig. 5 shows variation of SCF with change in G_{12}/E_1 where P=1Gpa, E_1=44.7 GPa, E_2/E_1=0.4 and μ_{12}=-0.3. For G_{12}/E_1=0.1 SCF increases with increase in polar angle and is maximum at 90^o. At G_{12}/E_1=0.1 to 0.3 SCF is minimum at θ=0^o whereas for G_{12}/E_1= 0.4 to 1.0 SCF is minimum at 90^o. For G_{12}/E_1=0.2 SCF is maximum at θ=55^o and for G_{12}/E_1= 0.3 to 1.0 SCF is maximum at θ=50^o. At θ=0^o and θ=90^o the SCF decreases monotonically with increase in $G_{12}/E_1$and at θ=45^o variation of SCF with increases in G_{12}/E_1 is negligible.

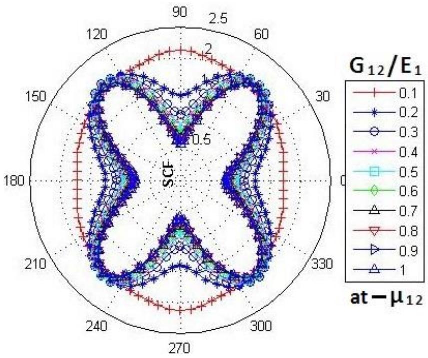

Fig. 5. SCF V/s θ for variousG_{12}/E_1at P=1Mpa, E_1=44.7 GPa, E_2/E_1=0.4 and μ_{12}=-0.3.

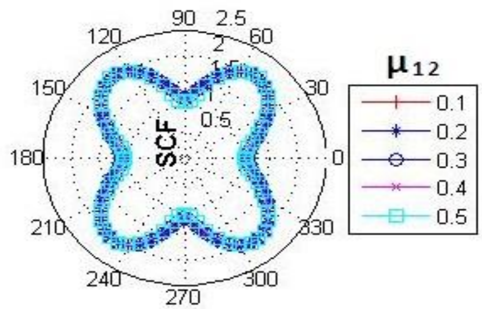

Fig. 6. SCF V/s θ for variousμ_{12}at P=1Mpa, E_1=44.7 GPa, E_2/E_1=0.4 and G_{12}/E_1=0.2.

Fig. 6 shows SCF distribution for various equidistant values of non-dimensional parameter μ_{12} for parameters P=1Gpa, E_1=44.7 GPa, E_2/E_1=0.4 and G_{12}/E_1=0.2. For all the values of μ_{12}starting from 0.1 to 0.5, SCF increases starting from 0^o polar angle up to 50^o, then decreases up to 90^o with minimum and maximum at 90^o and 50^o respectively.

Conclusion

Effect of variation of non-dimensional parameter on SCF of an orthotropic plate having circular cutout is investigated. Following conclusions are drawn from the study:

- At $\theta=0^o$ the SCF increases almost linearly with increase in E_2/E_1. At $\theta=45^o$ SCF increases from E_2/E_1=0.1 to 0.3 and after that decreases with increase in E_2/E_1. At $\theta=90^o$ SCF increases steeply with E_2/E_1
- At $\theta=0^o$ and $\theta=90^o$ the SCF decreases monotonically with increase in G_{12}/E_1. At $\theta=45^o$ variation of SCF with increases in G_{12}/E_1 is negligible.
- The most important conclusion drawn from the parametric analysis is that for orthotropic material (unlike isotropic materials) SCF depends on material properties. It increases with increase in E_2/E_1 ratio and decreases with increase in μ_{12} and G_{12}/E_1ratio.
- The non-dimensional analysis carried out here can be used as a reference for design of laminates.

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