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Geometrically Nonlinear Flexural Analysis of Rectangular Plates under Different Transverse Loadings Using Mesh-Free Method

Jeeoot Singh

Department of Mechanical Engineering, B.I.T., Mesra, Ranchi-835215, Jharkhand, India.

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ABSTRACT

The paper examines the large deformation flexural response of laminated composites and sandwich plates under various transverse loadings. The mathematical formulation of the actual physical problem of the plate subjected to mechanical loading is presented utilizing different shear deformation theories and von Karman nonlinear kinematics. The nonlinear governing differential equations of equilibrium are linearized using quadratic extrapolation technique. A meshless technique based on radial basis functions (RBFs) is used for analysis of the problems. Laminated and sandwich plates with immovable simply supported and clamped edges and subjected to point, line, sinusoidal and uniformly distributed loads are analyzed.

Introduction

Plates/panels are one of the important structural elements in aerospace, automotive, marine and other high performance engineering structures. During their service life they are subjected to different loading conditions and resulting deformations may be moderate to large. These structural components are preferably made up of fiber reinforced composites stacked in layers or sandwich constructions resulting in saving of weight.

Increase in the industrial use of laminated composites and sandwich construction necessitates the development of effective numerical tools/methods for the analysis of such structures. The large deformation analysis of the laminated composite plates subjected to different types of mechanical loading has been the subject of research interest of many investigators utilizing analytical and numerical tools. Using finite element method, Zhang and Yang [1] and Zhang and Kim [2] presented nonlinear analysis of thin to moderately thick laminated composite plates. Singh and Shukla [3]

presented the nonlinear bending response of laminated plates using radial basis function. Transverse bending of shear deformable laminated composite plates in Green-Lagrange sense accounting for the transverse shear and large rotations are presented by Dash and Singh [4]. A semi-analytical approach for the geometrically non-linear analysis of rectangular laminated plates with general in-plane and out-of-plane boundary conditions under a general distribution of out-of-plane loads is presented by Shufrin et al [5].

Most popular numerical techniques are based on versatile finite element. Finite difference, differential quadrature and boundary elements methods have also been used. In the last few years, a numerical tool that avoids the problem of mesh generation has gained momentum and has received attention of many researchers primarily due to the flexibility in the construction of finite dimensional sub-spaces. Particularly in case of extremely complex domain of interest, traditional numerical methods are somehow difficult to

- Corresponding author: **Jeeoot Singh**
- E-mail address: jeeootsingh@bitmesra.ac.in
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implement. Although most work to date on meshless methods using Radial Basis Functions (RBFs) relates to the scattered data approximation, there has recently been an increased interest in their use for solving partial differential equations (PDEs) in complex domain. This approach, which approximates the whole solution of the PDE directly using RBFs, is very attractive due to the fact that this is truly a mesh-free technique.

A mesh-free kp-Ritz method of solution based on the kernel particle approximation for the field variables is used for the large deflection flexural analysis of laminated composite plates by Liew et al [6]. Kansa [7] introduced the concept of solving PDEs using RBFs. Solution techniques based on RBFs are one of the best methods that have attracted several researchers in recent years especially in the area of computational mechanics [8-14]. A review of meshfree methods for laminated plate is presented by Liew et al [15]. Most of the works reported in literature using meshless methods are limited to the linear analysis only. Linear solution may be obtained with considerable ease and less computational cost when compared to nonlinear solutions. Literature shows a considerable gap for nonlinear analysis of laminated composite and sandwich plates by meshless methods using RBFs. Also, plates subjected to point and line loads undergoing moderate to large deformations are sparsely treated even by other numerical tools.

In the present study, the nonlinear flexural analysis of laminated composite and sandwich plates using different radial basis functions (RBFs) and various shear deformation theories is presented. Method of total linearization based on quadratic extrapolation technique is utilized in the present work. Laminated composite and sandwich plates with clamped and simply supported boundary conditions and subjected to uniform transverse pressure, sinusoidal pressure, line and point loads are analyzed.

Mathematical Formulation

A rectangular plate having a, b edge length along x, y axes respectively and thickness h along z axis whose mid plane is coinciding with xy plane of the coordinate system is considered. The geometry of rectangular laminated and sandwich plate in rectangular coordinate system is shown in Figures 1& 2 respectively. The displacement field at any point in the plate is expressed as [13]:

$$U = u_0(x, y) - z \frac{\partial w_0(x, y)}{\partial x} + f(z) \phi_x(x, y)$$

$$V = v_0(x, y) - z \frac{\partial w_0(x, y)}{\partial y} + f(z) \phi_y(x, y)$$
(1.1) (1.2)

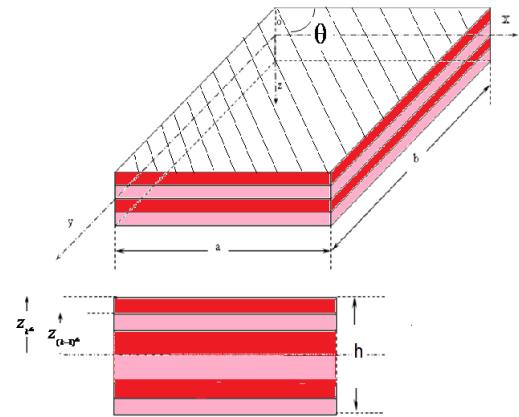
$$W = w_0(x, y)$$
(1.3)

Where, $f(z) = z \cdot \left(\frac{-2z^2}{h^2} \right)^{\ln(3)}$ is transverse shear stress function (TSF-1) proposed by Aydogdu [16] ,

$$f(z) = z \left(1 - \frac{4z^2}{3h^2} \right)$$

is transverse shear stress function

(TSF-2) proposed by Levinson [17] and $f(z) = z e^{-2(z/h)^2}$ is transverse shear stress function (TSF-3) proposed by Karama [18]. The parameters U, V and W are the in-plane and transverse displacements of the plate at any point (x, y, z) in x, y and z directions, respectively. u0, v0 and w0 are the displacements at mid plane of the plate at any point (x, y) in x, y and z directions, respectively. The functions Φ_x and Φ_y are the higher order rotations of the normal to the mid



plane due to shear deformation about y and x axes, respectively.

Fig. 1

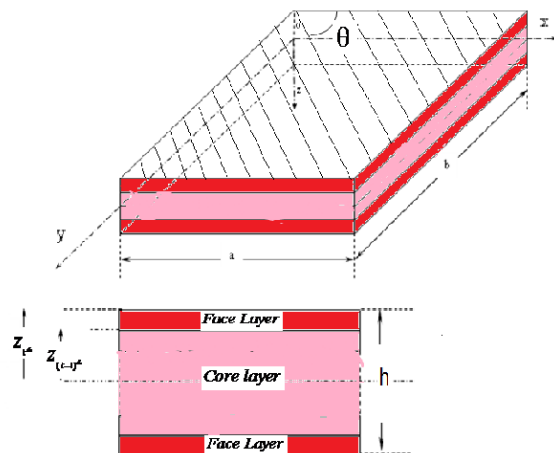


Fig. 2

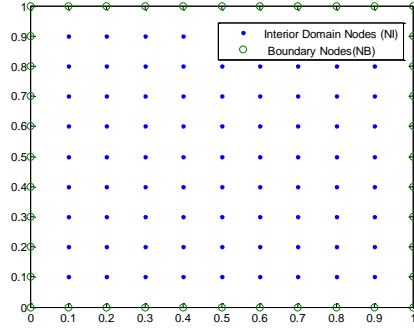


Fig. 3

von-Karman non-linear strain displacements relations are expressed as:

$$\left\{ \begin{array}{l} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{array} \right\} = \left\{ \begin{array}{l} \frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \\ \frac{\partial V}{\partial y} + \frac{1}{2} \left(\frac{\partial W}{\partial y} \right)^2 \\ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} + \left(\frac{\partial W}{\partial x} \right) \left(\frac{\partial W}{\partial y} \right) \\ \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \\ \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \end{array} \right\} \quad (2)$$

Assuming plane stress condition in a layer, the constitutive stress-strain relations for kth layer in the plate is expressed as:

$$\left\{ \begin{array}{l} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{array} \right\}_k = \left[\begin{array}{ccccc} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{array} \right]_k \left\{ \begin{array}{l} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{array} \right\}_k$$

Where, the parameters \bar{Q}_{ij} are the transformed reduced stiffness coefficients for layer.

The governing differential equations of plate are obtained using Hamilton's principle and expressed as:

$$\delta u : \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} - q_x = 0$$

$$\delta w : \left(\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) \left\{ \frac{\partial w_0}{\partial x} \right\} + \left(\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} \right) \left\{ \frac{\partial w_0}{\partial y} \right\} + N_{xx} \frac{\partial^2 w_0}{\partial x^2} + N_{yy} \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} - q_z = 0 \quad (4.3)$$

$$\delta \phi_x : \frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = 0$$

$$\delta \phi_y : \frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f = 0$$

The force and moment resultants in the plate and the detailed expressions of in-plane force and moment resultants and their derivatives used in equation is expressed by Singh and Shukla [3].

The boundary conditions for an arbitrary edge with simply supported and clamped edge conditions are:

(a) Simply Supported(SS):

$$x = 0 \text{ and } a : N_{xx} = 0, v_0 = 0, w_0 = 0, M_{xx} = 0, \phi_y = 0 \quad (5.1)$$

$$y = 0 \text{ and } b : u_0 = 0, N_{yy} = 0, w_0 = 0, \phi_x = 0, M_{yy} = 0 \quad (5.2)$$

(b) Clamped(CC) :

$$x = 0 \text{ and } a : u_0 = 0, v_0 = 0, w_0 = 0, \phi_x = 0, \phi_y = 0 \quad (5.3)$$

$$y = 0 \text{ and } b : u_0 = 0, v_0 = 0, w_0 = 0, \phi_x = 0, \phi_y = 0 \quad (5.4)$$

Solution Methodology

The governing differential equations (4.1 -4.5) are expressed in terms of displacement functions. A 2D rectangular domain having NB boundary nodes and ND interior nodes is shown in Fig.3 . The variable u_0 can be interpolated in form of radial distance between nodes. The solution of the coupled non linear governing differential equations (4.1-4.5) is assumed in terms of RBFs for nodes 1:N, as;

$$u_0 = \sum_{j=1}^N \alpha_j^u g(\|X - X_j\|, c)$$

$$v_0 = \sum_{j=1}^N \alpha_j^v g(\|X - X_j\|, c)$$

$$w_0 = \sum_{j=1}^N \alpha_j^w g(\|X - X_j\|, c)$$

$$\phi_x = \sum_{j=1}^N \alpha_j^{\phi_x} g(\|X - X_j\|, c)$$

$$\phi_y = \sum_{j=1}^N \alpha_j^{\phi_y} g(\|X - X_j\|, c)$$

Where, N is total numbers of nodes which is equal to the sum of boundary nodes NB and domain interior nodes ND. $g(\|X - X_j\|, c)$ is radial basis function, $\alpha_j^u, \alpha_j^v, \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ are unknown coefficients. $\|X - X_j\|$ is the radial distance between two nodes. The RBFs used in present solution methodology are expressed as:

RBF-1 :Thin plate spline

RBF-2

RBF-3 Polynomial function

$$g = r^c \quad c = 5$$

RBF-4 Multiquadrics function

$$g = (r^2 + c^2)^m \quad c = \alpha \sqrt{\left(\frac{a}{n_x}\right)^2 + \left(\frac{b}{n_y}\right)^2}$$

, m = 0.98

Where,

$$r = \|X - X_j\| = \sqrt{(x - x_j)^2 + (y - y_j)^2}$$

and m and c are shape parameters. a and b are the length and width of the plate, respectively. nx and ny are number of divisions along the length and width, respectively. α is a constant that governs the value of c for interior and boundary nodes.

The nonlinear governing differential equations are linearized using the quadratic extrapolation technique (Shukla and Nath [19]). Method of total linearization through quadratic extrapolation is used in the present work. Nonlinear terms are evaluated utilizing

quadratic extrapolation technique at every step of marching variable (load) and transferred to right hand side as pseudo load vector, updating the load vector. In this process the left hand side contains only linear terms. At first iteration of every step, nonlinear terms are evaluated using quadratic extrapolation scheme. In subsequent iterations at each step, functions are predicted as the mean of the values at current and preceding iterations. For first iteration, the predicted value Φ_j is extrapolated quadratically from the values of Φ at the three preceding steps as:

$$\Phi_j = \eta_1 \Phi_{j-1} + \eta_2 \Phi_{j-2} + \eta_3 \Phi_{j-3} \quad (6.3)$$

$$(10.1)$$

Where η_1, η_2 and η_3 are the coefficients of the quadratic extrapolation scheme and take the following values during the initial steps of the marching variables [19]:

$$\eta_1=1, \eta_2=0, \eta_3=0; \quad (6.4)$$

Results and Discussions:

A code in MATLAB is developed following the analysis procedure as discussed above. In order to demonstrate the accuracy and applicability of present formulation, several examples have been analyzed. Based on convergence study, 13x13 nodes are used throughout the study. The material properties are taken as:

$$\text{For laminated composite plate} \quad g = r^{2c} \log r \quad c = 3$$

Mat-1: E1=25E2 ; G12 = G13= 0.5E2 ; G23=0.2E2 ; $\nu_{12}=0.25$; a/h=10. Gaussian function

For sandwich plate

Mat-2: The properties of core are taken as [20];

E1 = 897949; E2 = 471424; G12 = 262 931;G13 = 159 914; G23 = 266 810; $\nu_{12} = 0.44$; a/h=10.

Skins material properties \bar{Q}_{skin} are related to

core properties \bar{Q}_{core} by a factor R as: (7.4)

$$\bar{Q}_{skin} = R \bar{Q}_{core}$$

The deflection, stress and the load parameters are non-dimensionalized as:

$$\text{For Mat-1} \quad \bar{w} = (w_c / h) \quad , \quad \bar{p} = \frac{q_0}{E_2} \left(\frac{a}{h}\right)^4$$

$$\text{For Mat-2} \quad \bar{w} = (w_c / h) \quad , \quad \bar{\sigma}_{xx} = \frac{\sigma_{xx}}{q_0}$$

$$\bar{\sigma}_{xz} = \frac{\sigma_{xz}}{q_0}$$

where W_c is central deflection, σ_{xx} and σ_{xz} are maximum stresses. q for different types of loading conditions (Figs-4(a)-4(g)) is defined as:

TYPE-1: Sinusoidal line load:

TYPE-2: Uniform line load:

TYPE-3: Point load:

TYPE-4: Sinusoidal load:

TYPE-5: Uniform load:

Here, N_I and N_x are total number of nodes in the domain and on the line, respectively and q_0 is maximum transverse pressure intensity.

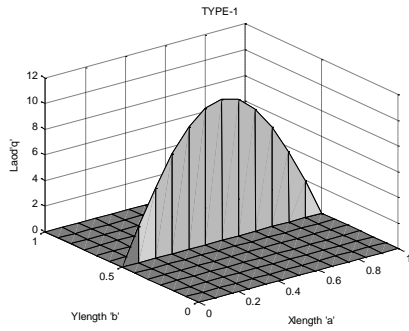


Fig. 4-(a)

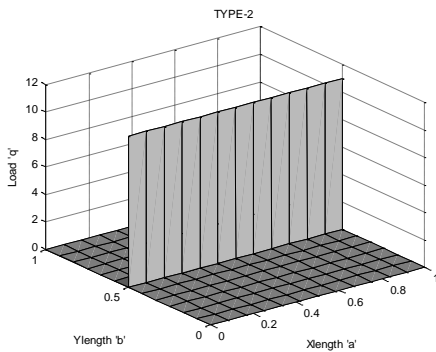


Fig. 4-(b)

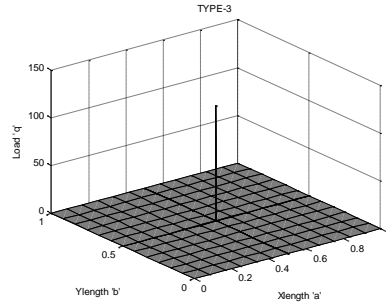


Fig. 4-(c)

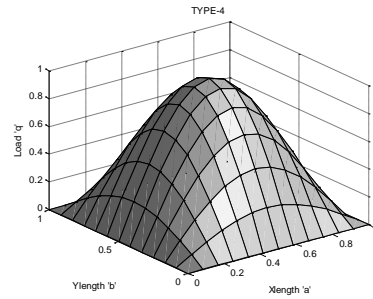


Fig.4-(d)

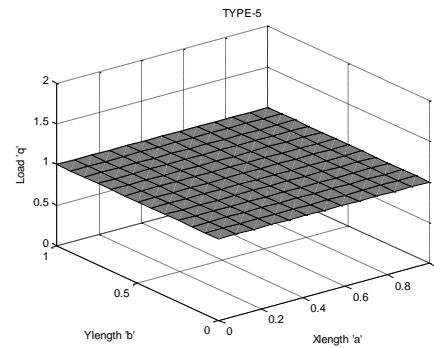


Fig. 4-(e)

4.1

In order to show the accuracy and efficiency of the present solution methodology, detailed convergence studies are carried out. The convergence for multiquadric RBF (RBF-4) becomes unstable after further increase of node from 13×13 for reported value of shape parameter

$(c = 1.5 / \sqrt{NS})$ by Xiang et al [13]. It is observed that multiquadrics RBFs with above shape parameter does not produce good convergence for point and line loads. Wang and Liu [8] have suggested that for $m=0.98$ and 1.03 , the RPIM-MQ performs the

$$q = (N_I / N_x).$$

$$q = (N_I / N_x).$$

$$q =$$

$$q =$$

best. For present analysis different values of shape parameters are tried and a suitable value of $m=0.98$, $\alpha = 0.47$ and 1.41 are used for clamped and simply supported boundary conditions, respectively throughout the analysis. It is found that if shape parameter c for boundary nodes is taken twice of that at interior nodes, the accuracy of results and convergence both increases. Other RBFs does not give satisfactory results for flexural analysis under clamped boundary conditions. The convergence of transverse central deflection of a simply supported (SS) isotropic square plate ($\nu = 0.3$) subjected to a concentrated load at center (TYPE-3) is carried out and shown in Fig.5. The deflection

\bar{w} is normalized as $\bar{w} = w_c D / qa^2$, where D is flexural rigidity and 'a' is plate length. It is observed that good convergence (within 3%) is obtained at 15×15 nodes. The present result is in good agreement with the results obtained by Timoshenko [21].

The convergence for the load TYPE-1, TYPE 2, TYPE - 3, TYPE- 4 and TYPE - 5 applied on sandwich plate (Mat-2, $a/h=10$, $h_{core} = 0.8h$) with all edges fixed is shown in Fig. 6. It can be seen that convergence for uniformly distributed load is faster and within 1% at 15×15 nodes. The convergence for point load is relatively slow and it converges within 3% at 15×15 nodes. The convergences of deflection corresponding to other load types are reasonably good.

4.2

Symmetric cross-ply [0/90/90/0] square plate with simply supported edges is analysed. The nonlinear transverse central deflection of the plate under uniformly distributed load (TYPE-5) utilizing different plate theories and radial basis functions is presented in Table-1. The results are also compared with results due to Kant and Kommineni [22] and Zhang and Kim [2]. The present results are found to be in good agreement. It can be seen that results obtained from all the RBFs except Gaussian radial basis function are in good agreement with analytical results due to Kant and Kommineni [22]. The Gaussian radial basis function underestimates the nonlinear response.

Linear and non-linear transverse central displacement of the plate subjected to various types of loadings are obtained and shown in Table 2. It can be seen that the central deflection is highest corresponding to load TYPE-3 followed by load TYPE-2 and is least for load TYPE-4. The nonlinear transverse displacement of the symmetric cross-ply plate with all edges clamped is also obtained and

shown in Fig.7. It can be seen that the central deflection is highest when the plate is subjected to point load (TYPE-3) at center and it is least when it is subjected to sinusoidal loading.

The analysis is further extended for a square sandwich plate (Mat-2, $a/h=10$). The thickness of top and bottom face is taken as $h_{face} = h/10$ and the thickness of the core sheet is taken as $h_{core} = 8h/10$. The effects of various transverse loading on central deflection of a clamped sandwich plate ($R=5$) is shown in Fig-8. It can be seen that the deflection is highest when plate is subjected to point load (TYPE-3) and least for sinusoidal load (TYPE-4). Comparison of non-linear transverse central deflection of simply supported sandwich square plate under different type of loadings is shown in Fig.9.

Fig. 10 shows the variation of nonlinear transverse deflection of mid plane along centerline of clamped sandwich plate under different transverse loads. It is observed that the deflection at center corresponding to point load is much higher as compared to other load types. This difference decreases at locations near the edges.

Nonlinear central deflections of simply supported square sandwich plate utilizing different radial basis functions (RBFs) are obtained and shown in Fig. 8. It can be seen that results obtained using Gaussian function (RBF-2) underestimates the nonlinear response as compared to other RBFs. Fig. 11 depicts the variation of nonlinear transverse central deflection of simply supported sandwich square plate with different core thickness. It can be seen that with increase in core thickness, the deflection increases, as expected.

Figs. 13-14 show the through thickness variation of stresses $\bar{\sigma}_{xx}$ and $\bar{\sigma}_{xz}$ respectively for a clamped square sandwich plate under different types of loads. It can be

seen that the stress $\bar{\sigma}_{xx}$ for line load (TYPE-2) is greater within the core, however this trends is not same in faces, where stress due to sinusoidal varying line load (TYPE-1) is higher as compared to line load (TYPE-2). The

variations of stress $\bar{\sigma}_{xz}$ follow the same trend inside the core and face where it is clear that maximum stress occur corresponding to line load (TYPE-2) and minimum for sinusoidal varying load (TYPE-4). Figs. 15-16 depict the effect of boundary conditions on non-

dimensional stresses $\bar{\sigma}_{xx}$ and $\bar{\sigma}_{xz}$, respectively of a square sandwich plate under point load at center (TYPE-3). It is seen that $\bar{\sigma}_{xx}$ is almost same for clamped and simply

supported plates whereas transverse shear stress $\bar{\sigma}_{xz}$ is more for simply supported sandwich plate.

Table-1 Nonlinear transverse central deflection (\bar{w}) of a simply supported [0/90/90/0] cross-ply square plate subjected to uniformly distributed load (Load TYPE-5), (Mat-1, TSF-2)

\bar{P}	Present						References			
	RBF4	RBF4	RBF4	RBF1	RBF2	RBF3	Kant [22]	Kant [22]	Zhang [2]	Zhang [2]
	THE-1	THE-2	THE-3	THE-3	THE-3	THE-3	Analytical	HOST	RDKQ-NL24	RDKQ-NL20
50	0.3387	0.3379	0.3393	0.3434	0.3258	0.3434	0.3560	0.3704	0.3704	0.3631
100	0.4954	0.4947	0.4960	0.4988	0.4665	0.4965	0.5100	0.5249	0.5249	0.5141
150	0.6033	0.6026	0.6039	0.6059	0.5610	0.6021	0.6100	0.6290	0.6290	0.6158
200	0.6874	0.6866	0.6881	0.6881	0.6456	0.6817	0.6890	0.7099	0.7099	0.6949
250	0.7421	0.7416	0.7423	0.7458	0.7120	0.7457	0.7470	0.7770	0.7770	0.7605

Table-2 Linear and non-linear transverse central deflection (\bar{w}) of a simply supported [0/90/90/0] cross-ply square Plate subjected to various types of loadings, (Mat-1, TSF-2, RBF-4)

\bar{P}	Linear					Nonlinear				
	TYPE-5	TYPE-4	TYPE-3	TYPE-2	TYPE-1	TYPE-5	TYPE-4	TYPE-3	TYPE-2	TYPE-1
10	0.10375	0.06853	0.31400	0.17475	0.14364	0.09891	0.06716	0.23716	0.15917	0.13450
15	0.15563	0.10280	0.47100	0.26213	0.21547	0.14133	0.09829	0.28637	0.21987	0.18903
20	0.20750	0.13707	0.62800	0.34950	0.28729	0.17885	0.12736	0.30777	0.26824	0.23456
25	0.25938	0.17133	0.78500	0.43688	0.35911	0.21213	0.15431	0.31539	0.30630	0.27202
30	0.31125	0.20560	0.94200	0.52425	0.43093	0.24189	0.17917	0.31688	0.33624	0.30287
40	0.41500	0.27414	1.25600	0.69900	0.57458	0.29353	0.22342		0.36958	0.34574
50	0.51875	0.34267	1.57000	0.87375	0.71822	0.33791	0.26191			0.36664
60	0.62250	0.41120	1.88400	1.04850	0.86187	0.37207	0.29594			

70	0.72625	0.47974	2.19800	1.22325	1.00551	0.40719	0.32637			
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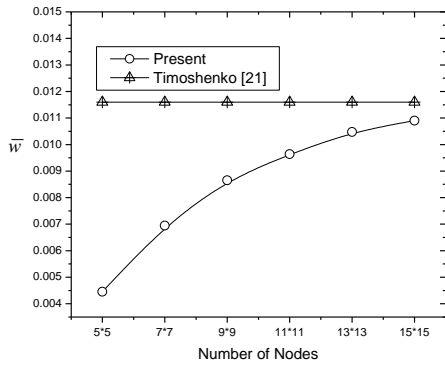


Fig. 5

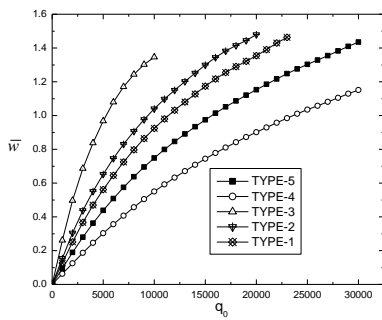


Fig. 6

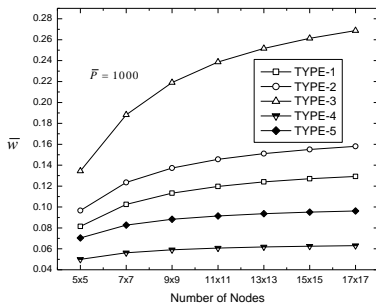


Fig. 7

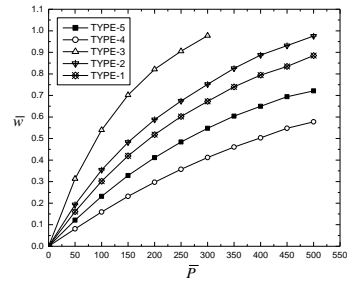


Fig.8.

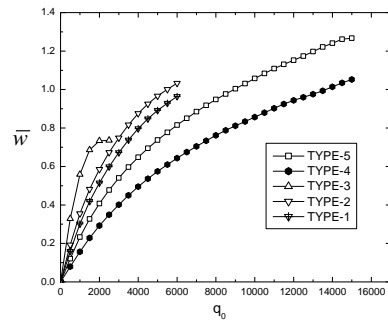


Fig. 9.

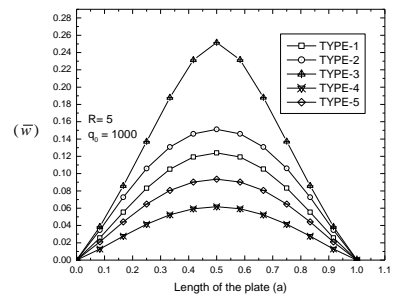


Fig.10:

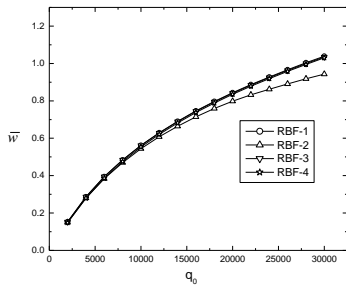


Fig. 11.

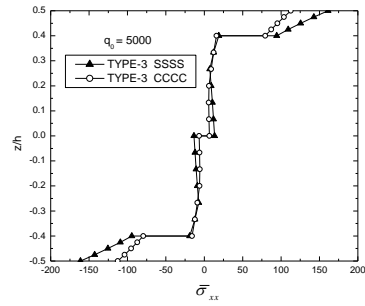


Fig. 15.

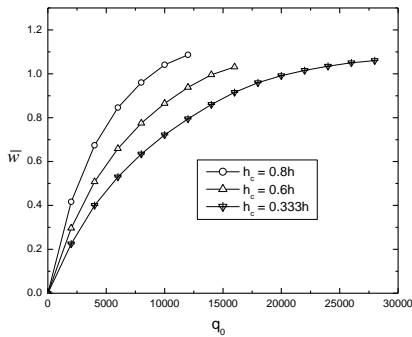


Fig. 12.

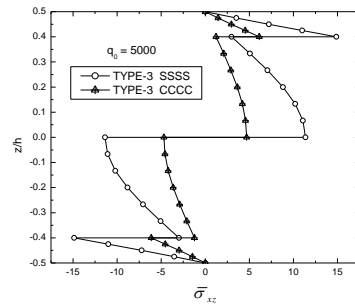


Fig. 16.

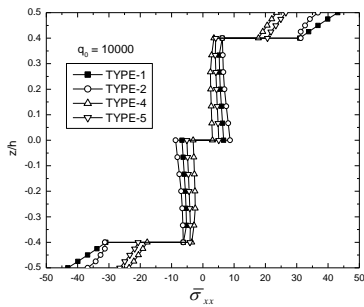


Fig. 13

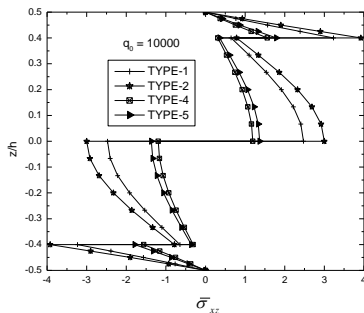


Fig. 14.

Conclusions

The nonlinear displacement response of laminated and sandwich composite plates subjected to different type of loadings are obtained using various RBFs and quadratic extrapolation technique. Effects of different loadings, core thickness and radial basis functions on nonlinear flexural responses are presented. It is observed that Gaussian function used as RBF underestimates the transverse deflection. The results presented herein show the applicability of present solution methodology. It is seen that selection of shape parameters plays important role in convergence of the solution. However, choice of shape parameter for better and faster convergence of deflection for the plate under point loads is still matter of further investigations.

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