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## Relating fractal dimension with roughness parameters in brass turning

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### ABSTRACT

The present work describes the evaluation of the conventional roughness parameters and fractal dimensions of the surface profiles generated by turning of brass with different machining conditions such as depth of cut, work-piece speed and feed rate. The surface roughness parameters on the generated-turned surfaces were measured with a stylus-type profilometer and processed through the dedicated advanced surface finish analysis software Talyprofile for evaluation of the roughness parameters and fractal dimension. Parameters used in the present investigation are: Vertical parameters –  $R_a$  (CLA,  $\mu\text{m}$ ) and  $R_q$  (RMS,  $\mu\text{m}$ ), Transverse parameters –  $R_{sm}$  (mean line peak spacing,  $\mu\text{m}$ ), Peak shape and amplitude parameters –  $R_{sk}$  (skewness),  $R_{ku}$  (kurtosis) and Fractal parameter –  $D$  (fractal dimension). The significant machining parameters for different roughness parameters and fractal dimension are found using ANOVA. The measured values of conventional roughness parameters and fractal dimension are correlated by non-linear regression analysis using fifth order polynomial model.

**Keywords:** ANOVA; Fractal dimension; Regression analysis; Roughness parameters; Turning;

### 1. Introduction

Turning is one of the common machining processes. However; it is well known that the process may induce damage at the machined surface. This damage may affect the functional properties of the material being turned. In a metal removal process such as turning, the surface generated consists of inherent irregularities left by tool, which are commonly defined as surface roughness. Such a surface is composed of a large number of length scales of superimposed roughness that are generally characterized by the standard deviation of surface peaks. Three statistical characteristics are generally used to describe the microstructure of machined surface topography: texture, waviness and roughness. The texture determines the anisotropic property of the surface. The waviness reflects the reference profile (or surface). The surface roughness is formed by the micro deformation during the machining process. The roughness influences physical phenomena, which are relevant to many engineering problems such as wear, contact resistance, wave scattering and pool boiling. Conventionally, three different types of parameters, viz.,

amplitude parameters, spacing parameters and hybrid parameters are used to characterize surface topography in general. Amplitude parameters are measures of the vertical characteristics of the surface deviations and examples of such parameters are centre line average roughness, root mean square roughness, skewness, kurtosis, peak-to-valley height etc. Spacing parameters are measures of the horizontal characteristics of the surface deviations and examples of such parameters are mean line peak spacing, high spot count, peak count etc. On the other hand, hybrid parameters are a combination of both the vertical and horizontal characteristics of the surface deviations and examples of such parameters are root mean square slope of profile, root mean square wavelength, core roughness depth, reduced peak height, valley depth, material ratio, peak area, valley area etc. Commonly used roughness parameters are centre line average roughness, root mean square roughness, mean peak line spacing, skewness and kurtosis. However, observations show that the deviation of a surface from its mean plane is a non-stationary random process [1]. Due to the multiscale nature of the surface, the variances and derivatives of surface peaks and other roughness parameters strongly depend on the resolution and the filter processing of measuring instruments. Hence, measuring instruments often provide different values on surface roughness. Ideally, rough surfaces should be characterized in such a way that the structural information of roughness at all scales is retained. To do so, quantifying the multiscale nature of surface roughness is essential. A unique property of a rough surface can be obtained by its scale-independent measurement. This requires the roughness reflecting the real surface structure at all magnifications. The similarity of a surface profile under different magnifications can be statistically characterized by fractal geometry since its topography is statistically self-affine. The ability to characterize surface roughness using scale-independent parameters is a specific feature of fractal approach. Fractal analysis provides information of the roughness at all length scales that exhibit fractal behavior. Based on Mandelbrot's work [2, 3], many researchers have attempted to describe and model rough surfaces using the fractal geometry theory [4-21]. Substantial investigation indicates that the surface topography has self-affine fractal characteristics [3, 6,

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9, 12]. Initially, Gagnepain and Roques-Carmes [4] approached the 3-D roughness surface using random walk noise and white noise. The fractal dimension was calculated using the Box counting method. Majumdar and Bhushan [5] simulated the machining surface based on the modified Weierstrass-Mandelbrot function, which is called the Majumdar–Bhushan function. Majumdar and Bhushan [5] thought that the self-affine surface could be characterised using two fractal parameters; namely the fractal dimension and the amplitude coefficient. Based on the fractal characteristics of a random Cantor set, Thomas and Krajcinovic [13] established a new model for elastic-perfectly plastic contact between surfaces. Until now, many fractal models have been developed and applied for surface simulation [6], elastic-plastic contact [7, 13], tribology [8], surface texture [9], adhesion [15], friction [16], wear [10, 11, 17] and so on.

In a material removal process, mechanical intervention happens over length scales, which extend from atomic dimensions to centimeters. The machine vibration, clearances and tolerances affect the outcome of the process at the largest of length scales (above  $10^{-3}$  m). The tool form, feed rate, tool radius in the case of single point cutting [22] and grit size in multiple point cutting [23], affect the process outcome at the intermediate length scales ( $10^{-6}$  to  $10^{-3}$  m). The roughness of the tool or details of the grit surfaces influence the final topography of the generated surface at the lowest length scales ( $10^{-9}$  to  $10^{-6}$  m). It has been shown that surfaces formed by electric discharge machining [21], cutting or grinding [9, 24], and worn surfaces [10, 11, 25] have fractal structures, and fractal parameters can reflect the intrinsic properties of surfaces to overcome the disadvantages of conventional roughness parameters. However, there is lack of information regarding the characteristics of roughness generated in turning particularly with respect to fractal dimension. Thus there is scope and need for further study in this respect. In the present work, the surface profiles generated by turning of brass are measured, digitized and processed to evaluate the statistical roughness parameters and fractal dimension. The relation between the fractal dimension and statistical roughness parameters are also investigated.

### 1. Fractal Dimension Calculation

Fractal calculation mainly includes the calculation of profile fractal dimension ( $1 < D < 2$ ) and the calculation of surface fractal dimension ( $2 < D < 3$ ) in tribological fractal research. Fractal calculation is generally involved with computer assisted image analysis of topography images in 2D or 3D of a surface obtained in analog or digital signals using profilometer or microscopy, etc. An effective method to convert these signals into the required data for calculating fractal dimensions must therefore be sought. Profile instruments can be used to obtain data in 2D, which are then directly used to calculate fractal dimension. The methods for calculating profile fractal dimension mainly include the yardstick, the box counting, the variation, the structure function and the power spectrum method [21].

The yardstick method employs the technique of ‘walking’ around a profile curve in a step length,  $r$ . A point on the profile curve is

chosen as a starting point of divider, whilst another point at a distance  $r$  from the starting point is taken as its end point. Repetitively, find the point-pair of dividers in the same way until the profile curve is entirely measured. Then, the summing up of the step lengths enables the curve length to be determined. The repetition of this calculation process at various step lengths allows all the curve length to be evaluated. Further, plotting of the curve lengths verses the step lengths on a log–log scale gives the slope  $m$  of a fitting line to be related to the fractal dimension  $D$  as  $D = 1 - m$ . It is possible that this method has abandoned some pivotal points, resulting in calculation error.

The principle of box counting method mainly involves an iteration operation to an initial square, whose area is supposed to be 1 and which covers the entire graph. The initial square is divided into four sub-squares and so on. After the  $n$  times operations, the number of sub-squares, which contain the discrete points of the profile graph are counted and the length  $L$  of the profile is approximately obtained. Then the fractal dimension is calculated as  $D = 1 + \log L / (n \cdot \log 2)$ .

The variation method has the advantage of being proven theoretically for all profiles (self-affine or not), and of giving quickly an estimation of the dimension of mathematical profiles. A well-known technique used to analyze surfaces consists in performing ‘slices’ through the surfaces, which allows one to transform a 3-D problem to 2-D. In other words, a surface is replaced by profiles, taken at different places, and the fractal dimension estimated over profiles is then related to the 3-D fractal dimension by the classical result: dimension of surface = 1 + dimension of profiles. Such a technique obviously decreases the problem size. Accurate results are hard to obtain for the surface dimension and the variation method gives the best approximations. The variation method algorithm is based on the local oscillation of the profile function  $Z$ .

The power spectrum method involves the evaluation of the power of the profile function. The modified Weierstrass–Mandelbort (W–M) function for a rough surface can be described as

$$z(x) = G^{(D-1)} \sum_{n=n_1}^{\alpha} \frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}} \quad (1)$$

where  $1 < D < 2$ ,  $\gamma > 1$ ,  $D$  is the fractal dimension;  $\gamma^n$  the discrete frequency spectrum of the surface roughness; and  $n_1$  the low cut-off frequency of the profile and  $G$  the characteristic length scale of the surface. The multi-scale nature of  $z(x)$  can be characterized by its power spectrum, which gives the amplitude of the roughness at all length scales. The parameters  $G$  and  $D$  can be found from the power spectrum of the W–M function by

$$S(\omega) = \frac{G^{2(D-1)}}{2 \ln \gamma} \frac{1}{\omega^{5-2D}} \quad (2)$$

where  $S(\omega)$  is the power of the spectrum, and  $\omega$  is the frequency of the surface roughness profile. Usually, the power law behavior would result in a straight line if  $S(\omega)$  is plotted as a function of  $\omega$  on a log–log graph. Using fast fourier transform (FFT), the power spectrum of profile can be calculated and then be plotted verses the frequency on a log–log scale. Thereafter, the fractal

dimension,  $D$ , can be related to the slope  $m$  of a fitting line on a log–log plot as:  $D = \frac{1}{2}(5 + m)$ .

The structure function method considers all points on the surface profile curve as a time sequence  $z(x)$  with fractal character. The structure function  $s(\tau)$  of sampling data on the profile curve can be described as

$$s(\tau) = [z(x + \tau) - z(x)]^2 = c\tau^{4-2D} \quad (3)$$

where  $[z(x+\tau)-z(x)]^2$  expresses the arithmetic average value of difference square, and  $\tau$  is the random choice value of data interval. Different  $\tau$  and the corresponding  $s(\tau)$  can be plotted verses the  $\tau$  on a log–log scale. Then, the fractal dimension  $D$  can be related to the slope  $m$  of a fitting line on log–log plot as:  $D = \frac{1}{2}(4 - m)$ .

## 2. Experimental Investigations

In the experimental study, machining tests were carried out for turning operation on a conventional lathe (Make - Mysore Kirloskar Ltd.). Workpiece material is brass. The workpiece employed were in the form of cylindrical bars about 50 mm long with an external diameter of 40 mm. During machining, no chatter, which would create significant vibrations, was identified. The workpiece speeds (rpm) chosen were 270, 421, 646 and 1000. Feed rates (mm/rev) selected were 0.028, 0.056, 0.084 and 0.113. Depths of cut (mm) taken were 0.10, 0.15, 0.20 and 0.25. Thus total 64 workpiece were turned for varying combinations of workpiece speed, feed rate and depth of cut. The surface roughness parameters on the generated-turned surfaces were measured with a portable stylus-type profilometer (Surtronic 3+, Taylor Hobson) equipped with a diamond stylus having tip radius 5  $\mu\text{m}$ . The profilometer was set to a cut-off length of 0.8 mm. The measurements on a single sample surface were repeated four times and the average value has been used as a data point. The measured profile is digitized and processed through the dedicated advanced surface finish analysis software Talyprofile for evaluation of the roughness parameters and fractal dimension. In this software fractal analysis is done using structure function method. Parameters used in the present investigation are: Vertical parameters –  $R_a$  (CLA,  $\mu\text{m}$ ) and  $R_q$  (RMS,  $\mu\text{m}$ ), Transverse parameters –  $R_{sm}$  (mean line peak spacing,  $\mu\text{m}$ ), Peak shape and amplitude parameters –  $R_{sk}$  (skewness),  $R_{ku}$  (kurtosis) and Fractal parameter –  $D$  (fractal dimension).

## 3. Analysis

In the present study surfaces are generated by turning of brass by varying the machining parameters with the motivation to study the effect of these machining parameters on surface roughness produced. The appropriate procedure for testing the significance of several parameters is the analysis of variance (ANOVA). The measured roughness parameters and fractal dimension are analyzed using ANOVA. Also, regression analysis is carried out to correlate fractal dimension with conventional roughness parameters.

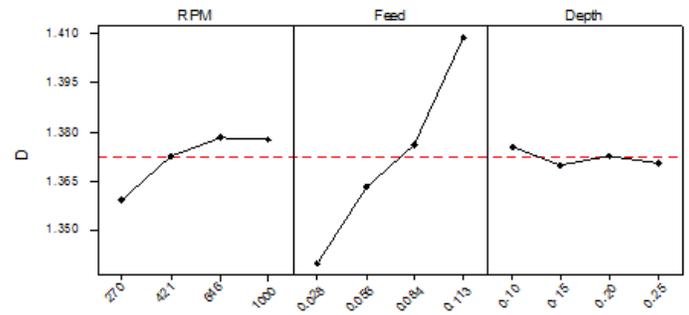


Fig. 1 Main effects plot for  $D$

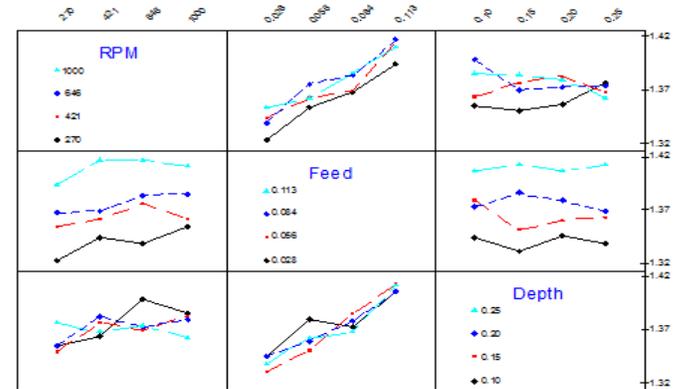


Fig. 2 Interaction plot for  $D$

### 3.1. ANOVA Tests

To determine the significant factors or factor interactions in affecting the surface roughness, measured roughness parameters and fractal dimension are tested in analysis of variance (ANOVA). The analysis is done with the help of MINITAB software (Release 13.1). ANOVA tables, main effects plots and full interaction plots are prepared for each parameter. Figures 1 and 2 show such typical main effects plot and interaction plots for fractal dimension. Such plots are prepared for each of the roughness parameters, but for the sake of brevity all such plots are not shown here. Actually, the main effect plot and the interaction plot appear to support each other for conclusion reached from ANOVA table. The ANOVA test is carried out based on 95% confidence level. Based on 95% confidence interval, RPM and feed rate have a statistically significant impact on fractal dimension, since their  $p$ -values are found to be smaller than 5% for turning of brass material though their interaction has no significant effect on fractal dimension. It is also observed that with increasing the feed rate and RPM, fractal dimension also increases. Only feed rate is the significant effect in determining the center line average value ( $R_a$ ) and root mean square roughness value ( $R_q$ ). Since roughness values increase with feed rate so to get smoother surface, lower feed rate should be chosen. For skewness, kurtosis and peak spacing, feed rate has the significant effect. While skewness and peak spacing increase with feed rate, kurtosis decreases with increasing the feed rate. Depth of cut has no statistical significant effect on roughness parameters and fractal dimension.

### 3.2. Regression Analysis

In any experiment, the experimenter is frequently interested in developing an interpolation equation for the response variable in the experiment. These equations are empirical models of the process that has been studied. The general approach to fitting empirical models is called regression analysis. In the present study, the measured values of conventional roughness parameters and fractal dimension for different processes and materials are correlated by non-linear regression analysis using fifth order polynomial model. The analysis has been carried out using MATLAB 6.5 as the computational platform. Figures 3-7 show the regression analysis results for different conventional roughness parameters with fractal dimension. The following empirical equations are obtained for the relation between fractal dimension and conventional roughness parameters used in the study.

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$$D = -0.032 \times R_a^5 + 0.32 \times R_a^4 - 1.2 \times R_a^3 + 2.2 \times R_a^2 - 1.8 \times R_a + 1.9 \quad (4)$$

$$D = -0.013 \times R_q^5 + 0.16 \times R_q^4 - 0.74 \times R_q^3 + 1.6 \times R_q^2 - 1.6 \times R_q + 1.9 \quad (5)$$

$$D = -0.0031 \times R_{sk}^5 - 0.28 \times R_{sk}^4 - 0.26 \times R_{sk}^3 + 0.053 \times R_{sk}^2 + 0.12 \times R_{sk} + 1.4 \quad (6)$$

$$D = -0.00068 \times R_{ku}^5 + 0.0028 \times R_{ku}^4 + 0.047 \times R_{ku}^3 - 0.37 \times R_{ku}^2 + 0.86 \times R_{ku} + 0.79 \quad (7)$$

$$D = 8.4 \times 10^6 \times R_{sm}^5 - 3.2 \times 10^6 \times R_{sm}^4 + 4.7 \times 10^5 \times R_{sm}^3 - 3.4 \times 10^4 \times R_{sm}^2 + 1.2 \times 10^3 \times R_{sm} - 15 \quad (8)$$

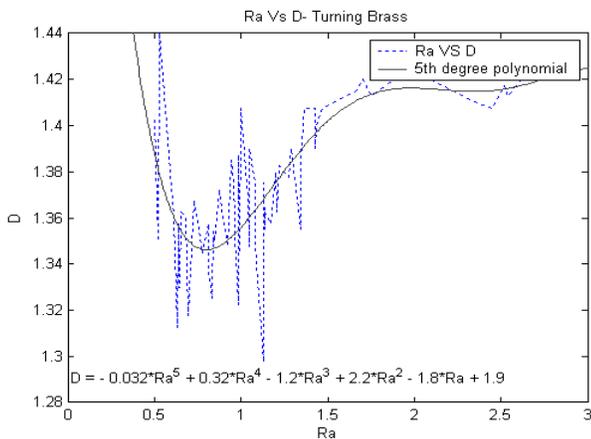


Fig. 3 Plot of  $R_a$  vs.  $D$  for turning of brass

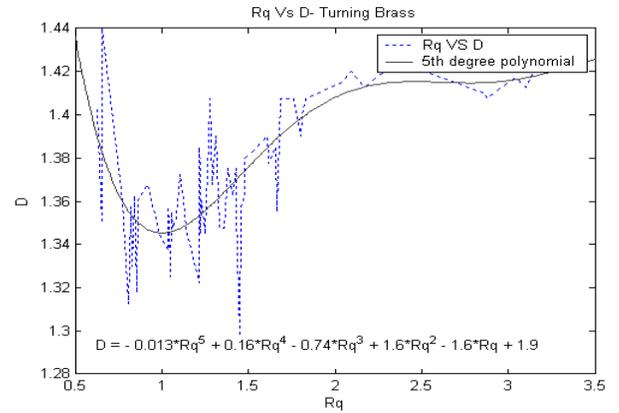


Fig. 4 Plot of  $R_q$  vs.  $D$  for turning of brass

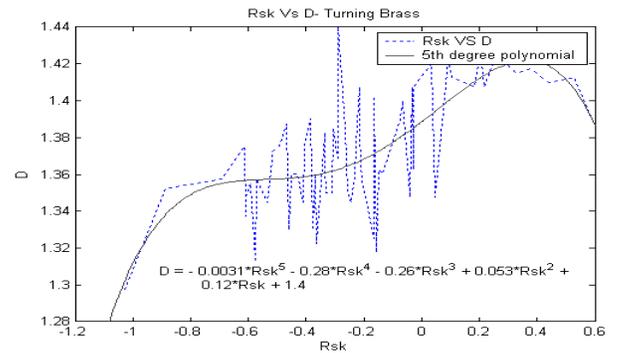


Fig. 5 Plot of  $R_{sk}$  vs.  $D$  for turning of brass

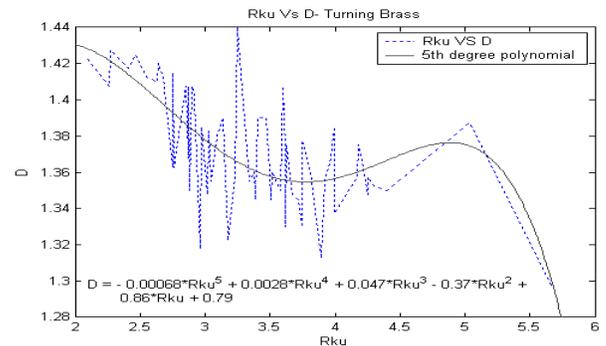


Fig. 6 Plot of  $R_{ku}$  vs.  $D$  for turning of brass

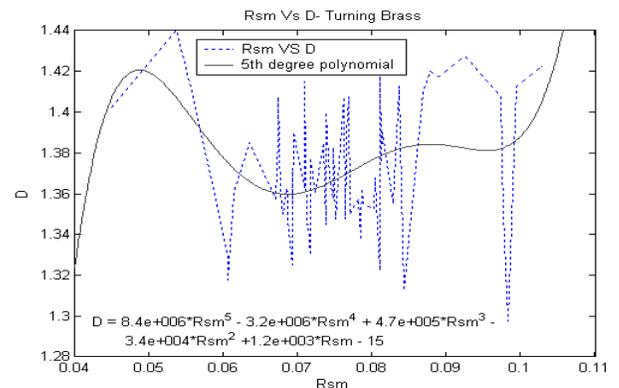


Fig. 7 Plot of  $R_{sm}$  vs.  $D$  for turning of brass

#### 4. Conclusion

In case of turning of brass, feed rate is the main affecting factor. All the roughness parameters including fractal dimension are controlled by feed rate. So, during turning operation of brass, feed rate should be controlled as per requirement of the surface finish. Depth of cut is seen to have no impact on surface roughness. So, to improve productivity higher range may be used if it does not worsen the surface microstructure of the material and the dimensional and geometric accuracy. In case of turning of brass, RPM has got some effect on fractal dimension in particular. In the present study, the measured values of conventional roughness parameters and fractal dimension for different processes and materials are correlated by non-linear regression analysis using fifth order polynomial model. However, more study is needed in this respect to have some generalized relation between fractal dimension and conventional roughness parameters.

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