Dynamic Analysis of 2R Manipulator With Joint Clearance

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A B S T R A C T

Generally Kinematic and Dynamic analysis of robots are performed without Joint clearance, because of which the results obtained are not accurate. For better analysis and control, the correct estimation and inclusion of joint effects is imminent. This project attempts to determine the effect of joint clearance on R-R type two link planar manipulator when the end effector moves in predetermined path (straight line). Joint and link are assumed to be rigid. Manipulator is modeled by using MSC Adams and joint clearance is incorporated to evaluate the contact forces by different methods. The joint clearances are varied from 0.01mm to 0.1mm and corresponding dynamic behavior is studied. Input to the model is taken so as to replicate human arm. The contact force during the contact process of revolute joint with clearance is one of the important contents of mechanical system. At the contact points normal and tangential forces, can be evaluated using different contact methods. Contact forces are plotted against time for different methods for various clearances. The obtained results are tabulated and compared to provide information on the clearance. Moreover I think this work will help the industry in correctly assessing the joint torques required for manipulating links with joint clearance.

Keywords: Joint clearance; Msc Adams; Contact force model

1. Introduction

Multi-link planar robot manipulators are general-purpose machines used for industrial automation in order to increase productivity, flexibility, and product quality. Other reasons for using industrial robots are cost saving, and elimination of hazardous and unpleasant work. Robot motion control is a key competence for robot manufacturers, and the current development is focused on increasing the robot performance, reducing the robot error (inaccurate), therefore there is a need to continuously improve the mathematical models in order to achieve increased performance and mechanical stiffness.

There are two types of primitive connections between a pair of links, as shown in the first is a prismatic joint where the pair of links makes a translational displacement along a fixed axis. In other words, one link slides on the other along a straight line. Therefore, it is also called a sliding joint. The second type of primitive joint is a revolute joint where a pair of links rotates about a fixed axis. This type of joint is often referred to as a hinge, articulated, or rotational joint.

1. Problem Statement

The objective of this paper is to evaluate the effect of joint clearance on the dynamic performance of the manipulators. As a case study, a 2 link manipulator is modelled using Msc Adams and it is made to move in a specified path (straight line). Contact is provided at both the joints by Kelvin voigt, Hertz’s, and Hunt crossley models with 0 to 0.1 mm clearance and second by varying the payloads (ML 1kg and 5kg). The effect on the contact forces generated is found and tabulated. Link parameters assumed throughout the paper are tabulated in Table 1.
## Table 1: Link Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the link (L1)</td>
<td>300 mm</td>
</tr>
<tr>
<td>Length of the link (L2)</td>
<td>400 mm</td>
</tr>
<tr>
<td>Width of the link (w)</td>
<td>40 mm</td>
</tr>
<tr>
<td>Radius of holes</td>
<td>10 mm</td>
</tr>
<tr>
<td>Depth of the cross section of link (d)</td>
<td>20 mm</td>
</tr>
<tr>
<td>Cross Section Area (A)</td>
<td>π x d mm²</td>
</tr>
<tr>
<td>Moment of inertia (I)</td>
<td>(π x d²)/12 mm⁴</td>
</tr>
<tr>
<td>Young's Modulus of the material (E)</td>
<td>2 x 10¹¹ N/mm²</td>
</tr>
<tr>
<td>Density (ρ)</td>
<td>1.3 g/cm³</td>
</tr>
<tr>
<td>Frictional mass (MP)</td>
<td>10.25N</td>
</tr>
</tbody>
</table>

2. Research Methodology

### A. CONTACT FORCES AND MOMENTS MODELS

The contact force and moment model during the contact process of a revolute joint with clearance is one of the important contents of mechanical system. At the contact points work normal and tangential forces, $F_N$ and $F_T$, respectively. These forces can be evaluated using a contact force law and a friction law, for example the classical laws (Coulomb, Hertz, etc.). The contributions to the generalized vector of forces and moments, are found by projecting the normal force and tangential forces onto the X and Y directions. These forces that act on the contact points of bodies $i$ and $j$ are transferred to the center of mass of bodies respectively, the forces and moments working on the center of mass of body $i$ are given by

$$F_i = F_N + F_T$$

$$M_i = - (y_1 - y_i^*)F_T + (x_1^* - x_i)F_T$$

The forces and moments working on the center of mass of body $j$ are given by

$$F_j = -F_i$$

$$M_j = - (y_j - y_j^*)F_T + (x_j^* - x_j)F_T$$

where $Q$ is its indicate the contact points on body $i$ and $j$.

#### 3.1 Spring Damper (Kelvin-Voigt Contact Force Model):

The simplest contact force relationship known as Kelvin-Voigt viscous-elastic model, is modeled by a parallel spring-damper element. The spring represents the elasticity of the contacting bodies, and the damper describes the loss of kinetic energy during the impact. This model assumes that both the spring and damper are linear. When the contact bodies are separating from each other, the energy loss is included in the contact model by multiplying the rebound force with a coefficient of restitution $e$.

The normal Kelvin-Voigt contact force $F_N$ is calculated from penetration depth $\delta$, multiplied by a spring constant $K$, yielding

$$F_N = \begin{cases} 
K\delta & \text{if } v_N > 0 \quad \text{(loading phase)} \\
K\delta e & \text{if } v_N < 0 \quad \text{(unloading phase)} 
\end{cases}$$

Where $K = \text{spring stiffness coefficient.}$

$\delta = \text{Represents elasticity deformation or the relative penetration depth.}$

$e = \text{The restitution coefficient.}$

$v_N = \text{The relative normal velocity of colliding bodies.}$

#### 3.2 Hertz’s Contact Force Model:

The best-known contact force law is due to the result of pioneering work by Hertz, which is based on the theory of elasticity. The Hertz contact theory is restricted to frictionless surfaces and perfectly elastic solids. This is a non-linear model but limited to impacts with elastic deformation and in its original form does not include damping. With this model, the contact process can be pictured as two rigid bodies interacting via a non-linear spring along the line of impact. The hypotheses used states that the deformation is concentrated in the vicinity of the contact area, elastic wave motion is neglected, and the total mass of each body moves with the velocity of its mass center.

The impact force is defined as [17]:

$$F_N = K\delta^n$$

Where $K$ and $n$ are constants, depending on material and geometric properties and computed by using elastostatic theory. The exponent $n$ is equal to 1.5 for metallic surfaces with circular and elliptical contacts. $K$ is the contact stiffness coefficient of the impact body, which is obtained from impact experiment of two spheres and valid for spherical contact only. $K$ is obtained from the following.

$$\sigma_j = \frac{1 - \nu^2}{\pi E_j}$$

Where $\nu$ and $E$ are Poisson ratio and Young’s modulus, respectively. $R_i$ and $R_j$ are radii of the two contact bodies, $\sigma_{ij}$ material parameters. Here by definition, the radius is negative for concave surfaces, such as for journal element, and positive for convex surfaces, such as for the bearing element. The advantage of Hertz contact model is that the geometric and material characteristics of the contacting surfaces are considered, which are important for dynamic characteristics analysis of contact. Besides, Hertz contact model is a non-linear relation between the penetration and the contact force. However, the Hertz contact model given by Eq. (4) is limited to contacts with elastic deformations and does not include energy dissipation.

#### 3.3 Hunt-Crossley Contact Force Model:

Although the Hertz law is based on the elasticity theory, some studies have been performed to extend the contact law to include energy dissipation. In fact, the most complicated part of modeling impacts is the process of energy transfer. If an elastic body is subjected to a cyclic load, the energy dissipation due to internal damping causes a hysteresis loop in the force-penetration diagram. Hunt and Crossley showed that the linear spring-damper model does not represent the physical nature of energy transferred during the impact. Instead, they represent the contact force by the Hertz forced penetration law with a non-linear viscous-elastic element. This approach is valid for direct central and frictionless impacts. The impact force model is defined as:

$$F_N = K\delta^n + b\delta^n\delta$$

Where
\[ b \] = The damping coefficient related to coefficient of restitution, \( c_0 \).

The advantage of Hunt-Crossley model, presented in Eq. (6), is based on Hertz law with a non-linear viscous elastic element, in which the energy dissipation is included. This contact force model represents the contact process as a non-linear spring-damper model along the direction of collision.

4. Simulation:

MSC ADAMS/View software is used to create static, kinematic and dynamic analysis of virtual prototypes by computer simulation. The link parameters assumed are tabulated in Table 1. A manipulator with two links and an end effector is modelled, by making the end effector to move in a straight line.

Contact:

Contact is provided by following below steps.

- In the contact type converter, select curve to curve.
- Select the journal_1 as the I Curve(s).
- Select the bearing_1 as the J Curve(s).
- The parameters change depending on the model and type of contact force.

The generated contact forces obtained from simulation with a clearance of 0.02 mm at joint_1 with 1 kg mass at End effector are presented in Fig. 4. The results obtained with different models (Kelvin-Voigt, Hertz's, and Hunt crossley) are presented in the graph. Here it is observed that the pattern of fluctuations is found to be same for all models. The Contact force is maximum at beginning as the impact is sudden and gradually decreased as it progressed and it is repeated for four cycles. The maximum contact forces generated by Kelvin-Voigt model then Hunt crossley and minimum contact force generated by Hertz's model.
The generated contact forces obtained from simulation with a clearance of 0.02mm at joint_2 with 1kg mass at End effector are presented in Fig.5. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. Here it is observed that the pattern of fluctuations is found to be same for all models. The contact force at joint 2 is observed to be more compared to joint 1. The Contact force is maximum at beginning as the impact is sudden and gradually decreased as it progressed and it is repeated for four cycles. The maximum contact forces generated by Hunt crossley model then Kelvin-Voigt and minimum contact force generated by Hertz’s model.

The generated contact forces obtained from simulation with a clearance of 0.1mm at joint_2 with 1kg mass at End effector are presented in Fig.7. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. Here it is observed that the pattern of fluctuations is found to be same for all models. As the clearance is increased from 0.02mm to 0.1mm, maximum contact force was increased. The maximum contact forces generated by Kelvin-Voigt model then Hunt crossley and Hertz’s model.

The generated contact forces obtained from simulation with a clearance of 0.02mm at joint_1 with 5kg mass at End effector are presented in Fig.8. Here it is observed that the pattern of fluctuations is found to be same for all models. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. The maximum contact forces generated by Hunt crossley model then Kelvin-Voigt and minimum contact force generated by Hertz’s model.

The generated contact forces obtained from simulation with a clearance of 0.02mm at joint_1 with 5kg mass at End effector are presented in Fig.9. Here it is observed that the pattern of fluctuations is found to be same for all models. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. The maximum contact forces generated by Hunt crossley model then Kelvin-Voigt and minimum contact force generated by Hertz’s model.
The generated contact forces obtained from simulation with a clearance of 0.1 mm at joint 1 with 5 kg mass at End effector are presented in Fig. 10. Here it is observed that the pattern of fluctuations is found to be same for all models. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. The maximum contact forces generated by Hunt crossley model then Kelvin-Voigt and minimum contact force generated by Hertz’s model.

The generated contact forces obtained from simulation with a clearance of 0.1 mm at joint 2 with 5 kg mass at End effector are presented in Fig. 11. Here it is observed that the pattern of fluctuations is found to be same for all models. The results obtained with different models (Kelvin-Voigt, Hertz’s, and Hunt crossley) are presented in the graph. The maximum contact forces generated by Kelvin-Voigt model then Hunt crossley and minimum contact force generated by Hertz model.

6. Conclusions

The simulation models for rigid link manipulators with joint clearance are presented. A two link rigid manipulator has been simulated in Msc Adams software and results are compared. The results of various impact models (Kelvin-voigt, Hertz’s, Hunt crossley) are compared. These models can predict the dynamic response of mechanisms and machines having unlubricated revolute joint clearance including the peak values of forces.

Joint 1

- When the End Effector moves in straight line, and as the joint clearance is varied from 0.02 mm to 0.1 mm with 1 kg as End effector mass, the contact force generated at the joint 1 is increased by 51.8% based on Kelvin-Voigt model, 167.2% based on Hertz’s model, 7.6% based on Hunt crossley model.

- When the End Effector moves in straight line, and as the joint clearance is varied from 0.02 mm to 0.1 mm with 5 kg as End effector mass, the contact force generated at the joint 1 is increased by 68.48% based on Kelvin-Voigt model, 73.7% based on Hertz’s model, 68.2% based on Hunt crossley model.

Joint 2

- When the End Effector moves in straight line, and as the joint clearance is varied from 0 to 0.1 mm with 1 kg as End effector mass, the contact force generated at the joint 2 is increased by 17.62% based on Kelvin-Voigt model, 3% based on Hertz’s model, 2.9% based on Hunt crossley model.

- When the End Effector moves in straight line, and as the joint clearance is varied from 0 to 0.1 mm with 5 kg as End effector mass, the contact force generated at the joint 2 is increased by 18.4% based on Kelvin-Voigt model, 5.3% based on Hertz’s model, and decreased by 13% based on Hunt crossley model.

- As clearance increased, power required to operate the system is found to increase drastically.

- The contact force transients are large initially as the impact is made suddenly, from then onwards there is a decrease in the contact forces generated as it behaves like continuous contact model.
7. References


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