Advanced Materials
Manufacturing & Characterization

journal home page: www.ijammc-griet.com

Experimental Modeling Of Meso Fractals Generated From Non Newtonian Fluid From Lifting Plate Hele-Shaw Cell

Bharatbhushan S. Kale[1*], Kiran Bhole[2], Prachi Khond[3]

1 Faculty of Mechanical Engineering, Datta Meghe College of Engineering, Airoli, Navi Mumbai, India
2 Faculty of Mechanical Engineering, Sardar Patel College of Engineering, Andheri, Mumbai, India
3 Faculty of Mechanical Engineering, New Horizon Institute of Technology & Management, Thane, Mumbai, Ind

A B S T R A C T

Various methods are existing to fabricate the meso and micro size fractals in the field of engineering. Viscous fingering also termed as Saffman Taylor instabilities is used in this research to get meso and micro fractals. Viscous fingering is obtained by allowing low viscous fluid to penetrate into high viscous fluid which is placed between two flat parallel plates separated by a small gap while lifting one plate in the z-direction away from another plate. The fluid flow between these two flat parallel plates is known as Hele-Shaw flow. The physics of fluid flow in Hele-Shaw cell & controlling factors for the development of viscous fingering are studied in this paper. The various controlling factors identified are the velocity of moving the plate, a gap between the plates, fluid quantity and fluid property: kinematic viscosity. A number of experiments were conducted with the identified controlling factor and the effect of these parameters on the shielding distance and radius of the circular plane were studied and analyzed. The mathematical model is developed using mathematics and fluid flow concept, which is responsible for the formation of micro fractals under various levels of control parameters. This mathematical model will help to find different attributes associated with micro fractals and to predict the pattern of viscous fingering. The fluid quantity and Reynolds number are supervising factors of a model.

Keywords: Saffman Taylor Instability; Hele Shaw flow; micro fractals; Mathematical modelling.

1. Introduction

The flow of fluids in a porous medium is a very important and interesting topic to study. This is interesting because of fluid flow exhibit various kinds of flow regime. These regimes are dependent on fluid surface interaction and properties of the fluid such as surface tension and viscosity. The shear gradient is also one of the important aspects to play in the developing fluid flow path regime. Modeling of such flow path regime is one of the challenging task due to multiple phenomenon governing the flow path development. The flow in the porous medium is governed by Darcy's law. Darcy's law is a phenomenological derived constitutive equation based on the experimental result. Darcy law is applicable to the Hele-Shaw cell (which is analogous to flow in a porous medium) [1]. The Darcy law for Hele Shaw cell for a Newtonian fluid is given by

$$u = -\frac{k}{\mu}\nabla p \qquad (1)$$

where u is the velocity of flow, k is the permeability of porous medium, μ kinematic viscosity, and ∇p dynamic pressure.

If $k=b^2/12$ then (where b is gap width between two plates) [2].

$$u = -\frac{b^2}{12}\nabla p \qquad (2)$$

The Darcy's law modified for non-Newtonian fluid is given below [3].

$$u = -\frac{b^2}{12\mu\left(\frac{|u|^2}{b^2}\right)}\nabla p, \qquad \nabla . u = 0 \qquad (3)$$

• Corresponing author Bharatbhushan S. Kale E-mail address: bhrtkale18@gmail.com
• Doi: http://dx.doi.org/10.11127/ijammc 2019.09.01

If the fluid is assumed as incompressible (∇.u=0), which reduces Darcy law to Laplacian equation [2] for the pressure in both fluids.

$$\nabla^2 p=0 \tag{4}$$

Hele-Shaw cell consists of two flat plates separated by a small distance in the z-direction. The dimension of plates in x and y-direction is much larger than z-direction. Out of these two plates, one plate is kept fixed and another plate can move in z-direction away from another plate or inclined to another plate. The Hele-Shaw cell is as shown in figure 1.

In the Hele-Shaw cell, a high viscous fluid is compressed by the pressing two plates together. A Low viscous fluid (in most cases air) is allowed to enter radially inwards by separating plates apart. Separation of plates create instability at the interface of fluids and developing micro fractals on both plates [see figure 2]. Microfracatals formed (also called viscous fingering) are the mirror image of each other. Here it is to be noted that, the fractal of high viscous fluid is seen on the bottom plate. These fractals are long wall type peaks. On the other hand mirror image of the same (long valley type path) is observed on the opposite plate. Actually, these valleys depict the path traced by the low viscosity fluid while separation.

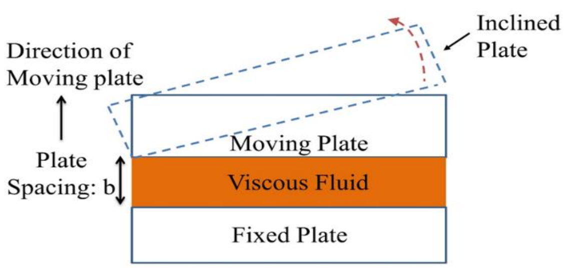

Figure 1(a): Schematic diagram of Hele Shaw Cell

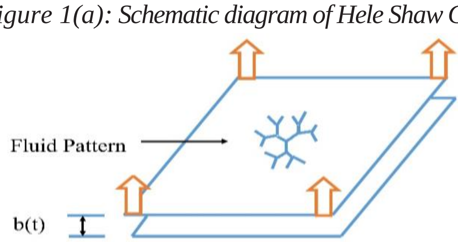

Figure 1(b): Micro Fractal Pattern

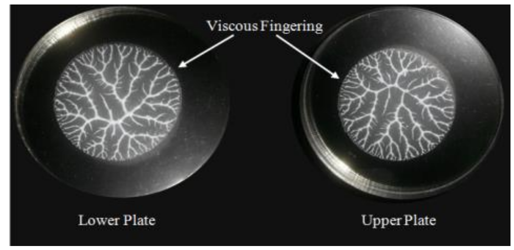

Figure 2: Viscous fingering in a Hele-shaw cell

Many researchers [3-15] have presented parameter based modified theory for Hele-Shaw cell to analyze fluid flow pattern. Out of these parameters, Inertia and viscous force are important parameters which play a vital role in the process of fingering formation. Because of these, Reynolds number is also important while analyzing the viscous fingering in Hele-Shaw cell as it is the ratio of inertia to viscous force. When high viscous fluid is used as compressing fluid , in that case, inertia force may be neglected, leading to very small Reynolds number (Re << 1). But in the case of dilute polymer solution whose viscosity is close to water, inertia force will become an utmost important factor which leads to Reynolds number more than unity [4]. Inertia force is also responsible for finger width. The finger width can increase with increasing fluid separation velocity. Many researchers [5-7] have neglected the viscosity of displacing fluid which causes the distribution of uniform pressure from all direction. By neglecting the viscosity of displacing fluid, a single parameter , the modified capillary number can help to determine finger shape and morphology.

$$Ca' = -\frac{U\mu}{\sigma}\left(\frac{w}{b}\right)^2 \tag{5}$$

Where σ is interfacial tension, U is the velocity of the finger, b the cell thickness, μ the viscosity of the displaced fluid, w the half Hele-Shaw cell width.

Studies [8-12] has simulated numerically the viscous fingering in the Hele-Shaw cell. In the process of fingering formation, gas bubble may get trapped between the Hele-Shaw cell [8]. A study investigated the Saffman-Taylor instability of a gas bubble expanding into a shear thinning liquid in a radial Hele-Shaw cell. Shear thinning is a characteristic of non-Newtonian fluids whose viscosity decreases with increasing shear stress. A correction factor was introduced in Newtonian instability of an expanding bubble in a radial cell for a weakly shear-thinning liquid. Simulation result has cleared that shear thinning liquid significantly controlled the viscous fingering. It prevents fingering from tip splitting [9]. The behavior of fingering viz; Shielding (one finger grows more in length by overpowering another finger) , spreading , splitting (side branching) and the effect of shear- thinning viscosity qualitatively predicted by simulating viscous fingering using FEM and VOF method. In fact, numerical simulation is an effective approach to study the behavior of fingering [10]. A study [11] derived the exact analytic solution of fingering structure with and without considering the surface tension. This exact solution gives an idea about the cusp formation and bubble fission in the viscous fingering formation process. The effect of surface tension was explored numerically on the bubble evolution and it was found that in the absence of surface tension it undergoes fission leading to the formation of two bubbles. A study [12] simulates numerically viscous fingering instabilities at high mobility-ratios. A mobility-ratio is defined as the ratio of the viscosity of the displaced fluid to the displacing fluid, which is considered as one of the factor in a study to investigate viscous fingering instability. Results of the numerical simulation showed that at high mobility ratio, mechanism of side branching, alternating side-branching, single-sided tip-splitting, double coalescence, and gradual coalescence and trailing lobe detachment dominant over the classical mechanism of viscous fingering.

Although, the behavior of the fingering, mechanism of fingering has been presented by researchers with the help of experimental, analytical and numerical simulation, but the exhaustive characterization of the process is still not presented considering various other parameters of the process. Influence of process parameters viz: quantity of fluid, a velocity of moving plate and gap between two plates are not yet studied and captured in the model exclusively. This paper presents an empirical model to capture formation and a fluid flow path of viscous fingering.

2. Methodology

2.1 Experimental characterization

The experiments are designed to study the effect of various parameters on the fractals formation. The three important parameters are considered for experimentation viz: quantity of fluid, gap between two plates and fluid separation velocity. These three identified factors are considered for design of experiments as shown in Table 1. Experiments were conducted in the combination of different levels for the characterization of the fractals. Micro fractals obtained from these experiments are studied and analyzed for mathematical modelling. The formed micro fractals are the resemblance of a Cayley tree up to three generations. Geometrical attributes like shielding distance, width of fingering, radius of a circular plane are defined and measured to quantify fractals formation. Shielding distance 'r_n' is the distance between two consecutive nodes of the branch (in mm) of formed fractals, 'R' is a radius of a circular plane of fluid and 'w' is the width of fluid.[see figure. 3 (f)]. This model has three generations, therefore three shielding distance r_1, r_2, r_3, and three width w_1, w_2, w_3, are measured.

The effect of various process parameters viz: fluid separation velocity, gap width and quantity of fluid on

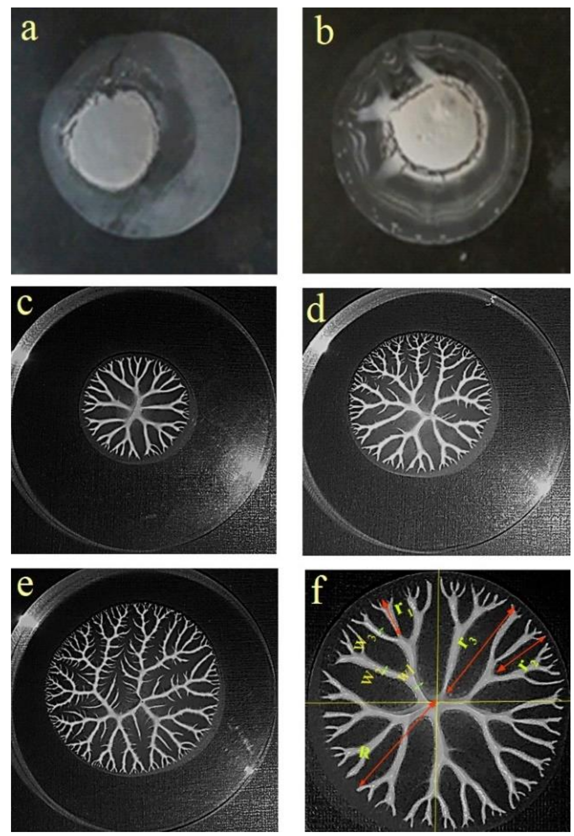

Figure 3. Final Formed Fractal Structures

(a) Q= 1.2 ml , V=2 mm/min , b=0.3 mm

(b) Q=1.2 ml, V=1.5 mm/min , b=0.2 mm

(c) Q = 1 ml, V = 1.3 mm/min, b = 0.2 mm

(d) Q = 1.2 ml, V = 1 mm/min b = 0.1 mm

(e) Q = 1 ml, V = 1.5 mm/min , b = 0.1 mm

(f) Micro fractal with geometrical attributes

Table 1: Design of experiments for the experimental characterization of the resin under Hele Shaw cell

Process Parameter	Levels		
Separation velocity (mm/min)	1	1.3	1.5
Gap Width (mm)	0.1	0.2	0.3
Quantity of fluid (ml)	0.8	1	1.2

viscous fingering is shown in figure 3. Formation of fractals depends upon the above mentioned parameters. For a large range of separation gap and separation velocity there is no formation of fractal which is shown in figure 3 (a) and (b). Lifting velocity and the gap width plays a vital role in finger formation. As the gap width goes on decreasing up to a certain limit, more fractals will form [see figure 3 (c) and (e)]. In fact to get the developed fingering pattern gap width should be kept minimum. When the gap width is higher, the fluid will not get fully compressed and hence the creep formation of fingering desired at the periphery of fluid will not initiate. Variation in the lifting velocity affects shielding distance r_3. Other than these two parameters the effect of quantity of fluid on viscous fingering is remarkable. The number of fingers and size of a pattern increases with fluid quantity as shown in figure 3 (c) and (d). But more fluid quantity beyond considered range is unable to form a pattern, it will just form fluid bob without any fractal formation.

2.2 Mathematical model

A mathematical model is presented in this paper is based on this analysis. Model is expressed in terms of shielding distance (r_n), Reynolds number (Re) and quantity of fluid (Q). Reynolds number is the ratio of the product of gap width and fluid separation velocity to the kinematic viscosity. Surface tension is considered a constant while defining model. Shielding distances (r_n), a radius of fingering pattern and numbers of fingers are the output of this model. The different correlations obtained are given in Equations (6) to (8).

$$r_n = A_1 + B_1\,Re + C_1\,Q + D_1 Re^2 + E_1 Re\,Q + F_1 Re^3 + G_1 Re^2\,Q \qquad (6)$$

$$R = A_2 + B_2\,Re + C_2\,Q + D_2 Re^2 + E_2 Re\,Q + F_2 Re^3 + G_2 Re^2\,Q \qquad (7)$$

$$N = A_3 + B_3\,Re + C_3\,Q + D_3 Re^2 + E_3 Re\,Q + F_3 Q^2 + G_3 Re^3 + H_3 Re^2 Q + I_3 Re\,Q^2 + J_3 Re^4 + K_3 Re^3 Q + L_3 Re^2 Q^2 \qquad (8)$$

Where $A_1, A_2, B_1, B_2 \ldots$ are constants.

The presented model is accurate up to 85 %. The model values obtained are within considerable limit. This model can be used to find out the pattern of the viscous fingering. Fluid properties viz. kinematic viscosity, surface tension are the important parameters for the growth of fingering. In the derived model, only kinematic viscosity is considered. Hence this model is biased to kinematic viscosity. Kinematic viscosity, gap width, fluid separation velocity and fluid quantity these input parameters are considered to define the model. The different correlations found out from the experiments are given in Equations (9) to (13), shown below.

$$r_1 = 0.06563 + 1.076\,e + 07\,Re + 3.288\,e + 06\,Q - 1.235e + 13\,Re^2 - 6.739e + 12\,Re\,Q + 3.718e + 18\,Re^3 + 4.286e + 18\,Re^2\,Q \qquad (9)$$

$$r_2 = 9.931 + 2.246e + 07Re + 1.831e + 07Q - 2.872e + 12\,Re^2 - 3.489e + 13\,Re\,Q - 5.368e + 18\,Re^3 + 1.708e + 19\,Re^2\,Q \qquad (10)$$

$$r_3 = 0.06563 + 1.076\,e + 07\,Re + 3.288\,e + 06\,Q - 1.235e + 13\,Re^2 - 6.739e + 12\,Re\,Q + 3.718e + 18\,Re^3 + 4.286e + 18\,Re^2\,Q \qquad (11)$$

$$R = 0.2101 + 4.863e + 07\,Re + 3.587e + 06Q - 7.004e + 13Re^2 + 1.421e + 13\,Re\,Q + 2.464e + 19Re^3 - 3.424e + 18Re^2\,Q \qquad (12)$$

$$N = -3559 + 1.071\,e + 10\,Re + 5.131\,e + 09Q - 1.226\,e + 16Re^2 - 9.95\,e + 15Re\,Q - 2.808\,e + 15Q^2 + 7.54\,e + 21Re^3 + 3.615e + 21Re^2Q + 5.81e + 21Re\,Q^2 - 2.084e + 27Re^4 + 6.255e + 26\,Re^3Q + -2.73e + 27Re^2Q^2 \qquad (13)$$

3. Results and Discussion

The effect of process parameters viz: velocity of moving the plate, a gap between the plates, fluid quantity and fluid property: kinematic viscosity on the viscous fingering is studied. The derived attributes of the viscous fingering pattern from the model are compared with the experimental values. The results obtained from the experiments are studied and analyzed.

Figure 4 and 5 are the representation of the effect of gap width on the shielding distance (r_n) and radius of circular plane (R) respectively. The graphs are linear in nature. The shielding distance and radius of a circular plane are inversely proportional to the gap width. At a lower value of gap width the shielding distances and radius are clear and distinct. Shielding distance and radius decreases with increasing value of gap width. But this will happen within a certain limit, beyond that limit the fingering pattern will not get formed. Hence minimum gap width is essential to get the fully developed fingering pattern.

The model values of shielding distance are closed to the experimental value. The shielding distances r_1 and r_2 obtained from the model are almost identical with the experimental value. But in case of shielding distance r_3, the error is more compared to the r_1 and r_2. It is because of surface tension is not taken into consideration. It is assumed as constant.

Model for a radius of a circular plane seems to be accurate, as the experimental and model values are equivalent to each other.

Figure 6 and 7 shows the effect of fluid quantity on the radius of a circular plane (R) and shielding distance (r_n) respectively. It is quite obvious that more quantity of fluid gives a bigger size of compressed fluid. Hence radius of a

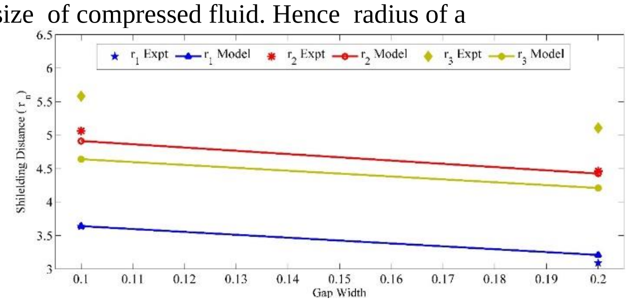

Figure 4: Effect of Gap Width on Shielding distance

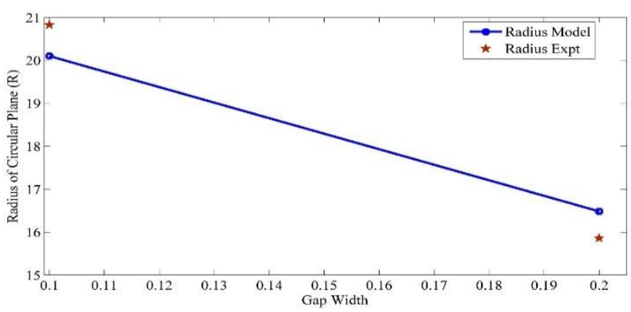

Figure 5: Effect of Gap Width on Radius of circular

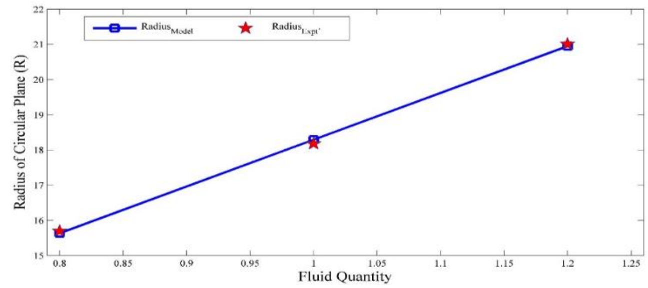

Figure 6: Effect of Fluid Quantity on Radius of circular plane

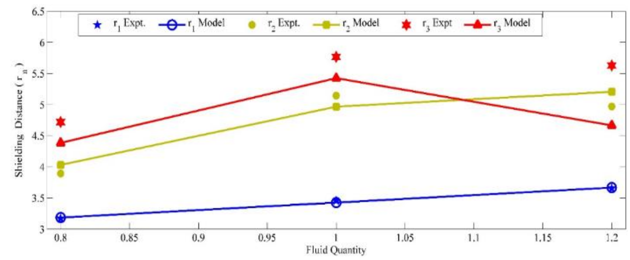

Figure 7: Effect of Fluid Quantity on Shielding distance

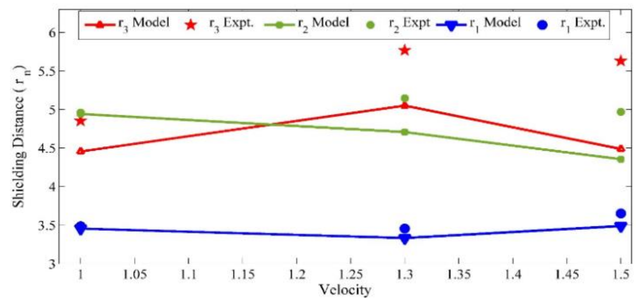

Figure 8: Effect of velocity on Shielding distance.

circular plane linearly increases with increasing fluid quantity. The defined mathematical model seems to be very accurate with the experimental value as shown in figure 6.

Shielding distance r_1 and r_2 is linearly proportional to fluid quantity like radius (R). But shielding distance r_3 increases with increasing fluid quantity up to certain limit, beyond that r_3 keeps on decreases. A mathematical model gives the accurate values of r_1 and r_2 with minimum error. But the difference between the model value and experimental values of r_3 is more compared to r_1 and r_2.

Figure 8 shows the transformation of shielding distance with respect to lifting velocity. From the figure it is seen that shielding distance r_1 and r_2 are almost constant with change in velocity. But r_3 is continuously varying according to velocity because r_3 is the first shielding distance which initiates due to the lifting of an upper plate. When the upper plate of the Hele-Shaw cell starts moving away from the lower plate with a certain velocity, air starts entering radially from the outer periphery of the circular plate. And this entered air initiates the formation of fingering. Hence the speed of the entering air has major importance in the formation of r_3 as compared to r_1 and r_2, as these shielding distances are the innermost part of the fingering. The r_1 and r_2 derived from the model and the experiments are nearly similar in values. But again the model value of r_3 is different from the experimental values.

The velocity of an upper plate does not affect the radius of a circular plane, as the upper plate moves after the compression of the high viscous fluid.

4. Conclusion

In this paper, we have experimentally studied the fluid flow in a Hele-Shaw cell and the controlling factor for the development of viscous fingering. A Darcy's law was stated and explained for Newtonian and non- Newtonian fluid. We identified the process parameters viz. fluid separation velocity, gap width, fluid quantity, and kinematic viscosity and their levels to study and analysis viscous fingering in lifting plate Hele-Shaw cell. We studied exclusive micro fractals formation resulting from various levels of process parameters using mathematics and fluid flow concept to develop a mathematical model. A mathematical model was developed to know the values of shielding distances, a radius of circular plane, and the number of fingers. The fluid quantity and Reynolds number are the supervising factors of the model. The effect of process parameters on shielding distances and the radius of a circular plane was studied. The gap width is inversely proportional to the shielding distances and radius. Hence to get the distinct structure, gap width should be minimum. The radius of a circular plane and shielding distances is linearly varying with the fluid quantity and shows a little variation with respect to velocity.

The parameters obtained from the developed model are matching with the experimental values. But in case of shielding distances r_3 the model shows the variation with respect to the experimental values. As the upper plate of Hele-Shaw cell moves in the upward direction and low viscous fluid: air comes in direct contact with the high viscous fluid the shielding distance r_3 forms simultaneously. A speed of the air penetrating the high viscous fluid is also an important factor in fingering formation and fingering formation starts with the initiation of shielding distance r_3. Hence there is variation in the experimental and model values of r_3.

The effect of the process parameters on the process of fingering formation was studied through this paper. The developed model is accurate up to 85 % with the experimental values and it can be used to predict the pattern of viscous fingering.

Acknowledgment
Authors would like to acknowledge financial support for this work through Minor Research Grant, University of Mumbai

References

[1] P. G. Saffman, 1986, "Viscous fingering in Hele-Shaw cells," Journal of Fluid Mechanics, 173(1), pp. 73-94.
[2] X. Guan and R. Pitchumani, 2003, "Viscous Fingering in a Hele-Shaw Cell With Finite Viscosity Ratio and Interfacial Tension," Journal of Fluids Engineering, 125(2), pp. 354-364.
[3] L. Kondic, P. Palffy-Muhoray, and M. J. Shelley, 1996,"Models of non-Newtonian Hele-Shaw flow," Physical Review E, 54(5), pp. R4536-R4539.
[4] Christophe Chevalier, Martin E. Ben Amar, Daniel Bonn, and Anke Lindner,2006, "Inertial effects on Saffman–Taylor viscous fingering," Journal of Fluid Mechanics, 552, pp. 83–97.
[5] C. W. Park and G. M. Homsy, 1985, "The instability of long fingers in Hele–Shaw flows," Physics of Fluids, 28(6), pp. 1583-1585.
[6] T. Maxworthy, 1987, "The nonlinear growth of a gravitationally unstable interface in a Hele-Shaw cell," Journal of Fluid Mechanics, 177(1), pp. 207-232.
[7] A. R. Kopf-Sill and G. M. Homsy, 1987, "Narrow fingers in a Hele–Shaw cell," Physics of Fluids, 30(9), pp. 2607–2609.
[8] L. Kondic, M. J. Shelley, and P. Palffy-Muhoray, 1998, "Non-Newtonian Hele-Shaw Flow and the Saffman-Taylor Instability," Physical Review Letters, 80(7), pp. 1433–1436.
[9] T. Yamamoto, H. Kamikawa, N. Mori, and K. Nakamura, 2002, "Numerical Simulation of Viscous Fingering in Non-Newtonian Fluids in a Hele-Shaw Cell." Journal of the Society of Rheology, Japan, 30(3), pp. 121–127.
[10] M. J. Shelley, F.-R. Tian, and K. Wlodarski, 1997, "Hele - Shaw flow and pattern formation in a time-dependent gap," Nonlinearity, 10(6), pp. 1471–1495.
[11] Sesini, P.A., de Souza, D.A. and Coutinho, A.L., 2010, "Finite element simulation of viscous fingering in miscible displacements at high mobility-ratios "Journal of the Brazilian Society of Mechanical Sciences and Engineering, 32(3), pp. 292-299.
[12] R. Teodorescu, P. Wiegmann, and A. Zabrodin, 2008, "Fingering patterns in Hele-Shaw flows are density shock wave solutions of dispersionless KdV hierarchy," Physical Review Letters, 08, pp. 07363-07369.
[13] H. Hadavinia, S. Advani, and R. Fenner,1995,"The evolution of radial fingering in a Hele-Shaw cell using C1 continuous overhauser boundary element method," Engineering Analysis with Boundary Elements, 16(2), pp. 183–195.
[14] S. K. Kabiraj and S. Tarafdar, 2003, "Finger velocities in the lifting Hele–Shaw cell," Physical A: Statistical Mechanics and its Applications, 328(3-4), pp. 305–314.

[15] S. K. Kabiraj and S. Tarafdar, "Finger velocities in the lifting Hele–Shaw cell," Physica A: Statistical Mechanics and its Applications, vol. 328, no. 3-4, pp. 305–314, 2003.
[16] J. Miranda and M. Widom, 1998, "Radial fingering in a Hele-Shaw cell: a weakly nonlinear analysis," Physica D: Nonlinear Phenomena, 120(3-4), pp. 315–328.
[17] R. Brandão, J. V. Fontana, and J. A. Miranda, 2013, "Stretch flow of confined non-Newtonian fluids: Nonlinear fingering dynamics," Physical Review E, 88(6), pp. 063003.
[18] Lindner, A., Bonn, D., Poiré, E.C., Amar, M.B. and Meunier, J., 2002, "Viscous fingering in non-Newtonian fluids." Journal of Fluid Mechanics, 469, pp.237-256.
[19] Holzbecher, E., 2009, "Modeling of viscous fingering." In COMSOL conference, Milan.
[20] Cummings, L.J., Howison, S.D. and King, J.R., 1999, "Two-dimensional Stokes and Hele-Shaw flows with free surfaces." European Journal of Applied Mathematics, 10(6), pp.635-680.
[21] K. Imomnazarov, 2004, "Modified Darcy laws for conducting porous media," Mathematical and Computer Modelling, 40(1-2), pp. 5–10.
[22] Fontana, J.V., Dias, E.O. and Miranda, J.A., 2014, "Controlling and minimizing fingering instabilities in non-Newtonian fluids." Physical Review E, 89, pp. 013016.
[23] T. U. Islam and P. S. Gandhi, 2016,"Fabrication of Multscale Fractal-Like Structures by Controlling Fluid Interface Instability," Scientific Reports, 6(1).
[24] A. A. Singare, B. S. Kale, and K. S. Bhole, 2018, "Experimental Characterization of Meso-Micro Fractals from Nanoparticle Seeded Resin in Lifting Plate Hele-Shaw Cell," Materials Today: Proceedings, 5(11), pp. 24213–24220.